Name: ______________________________

Math 1A Quiz 2
September 9, 2008
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You have until 4:30 to complete this quiz. You must show your work.

1. (3 pts) The graph of a function $f(x)$ is pictured below. Find each of the following, or state that they do not exist:

   (a) $\lim_{x\to 0} f(x) = 0$

   (b) $\lim_{x\to -2^+} f(x) =$ -1

   (c) $\lim_{x\to 1} f(x)$ DNE

   (d) $\lim_{x\to 1^-} f(x) =$1

   (e) $\lim_{x\to 2} f(x) = 1$

   (f) $\lim_{x\to -1} f(x) = 0$

2. (3 pts) Prove, using the $\epsilon - \delta$ definition of a limit, that $\lim_{x\to -3} 2x - 1 = -7$

   Let $\epsilon > 0$ be given. We want to find $\delta > 0$ such that $0 < |x+3| < \delta \to |2x - 1 + 7| < \epsilon$

   So we want $|2x + 6| < \epsilon$, which is the same as $2|x+3| < \epsilon$, which happens if $|x+3| < \epsilon/2$

   Claim: $\delta = \epsilon/2$ works

   Proof:

$$\begin{aligned} |x+3| < \delta = \epsilon/2 \\ 2|x+3| < \epsilon \\ |2x+6| < \epsilon \end{aligned}$$

3. (4 pts) Prove, using the $\epsilon - \delta$ definition of a limit, that $\lim_{x\to 16} \sqrt{x} = 4$

   Let $\epsilon > 0$ be given. We want to find $\delta > 0$ st $0 < |x - 16| < \delta \Rightarrow |\sqrt{x} - 4| < \epsilon$

$$\begin{aligned} |\sqrt{x} - 4| &= |\sqrt{x} - 4|\frac{|\sqrt{x}+4|}{|\sqrt{x}+4|} \\ &= \frac{|x-16|}{\sqrt{x}+4} \end{aligned}$$

   Note also that if $|x - 16| < 1$, then $15 < x < 17$, so $\sqrt{15} + 4 < \sqrt{x} + 4 < \sqrt{17} + 4$. Since the $\sqrt{x} + 4$ is in the denominator, we'll replace it by the smaller thing:

$$\frac{|x-16|}{\sqrt{x}+4} < \frac{|x-16|}{\sqrt{15}+4}$$

   We want this to be $< \epsilon$, so we'll guess $\delta = \min(1, (\sqrt{15} + 4)\epsilon)$

   Claim: This works

   Proof: If $|x - 16| < \delta = \min(1, (\sqrt{15} + 4)\epsilon)$, then $|x - 16| < 1$ and $|x - 16| < (\sqrt{15} + 4)\epsilon)$. Be previous work, we know $\sqrt{x} + 4 > \sqrt{15} + 4$ under these conditions, so we can do the following:

$$
\begin{aligned}
|\sqrt{x} - 4| &= |\sqrt{x} - 4| \frac{|\sqrt{x} + 4|}{|\sqrt{x} + 4|} \\
&= \frac{|x - 16|}{\sqrt{x} + 4} \\
&< \frac{|x - 16|}{\sqrt{15} + 4} \\
&< \frac{(\sqrt{15} + 4)\epsilon}{\sqrt{15} + 4} = \epsilon
\end{aligned}
$$

as was to be shown