

Below is (one version of) the midterm Prof. Hald gave last fall. No guarantees this year's will be anything similar, but I know some of you wanted it.

Basic Rules

1. Differentiate each of the following. Do not simplify your results.

(a) $x^4 - \frac{1}{x} + 3 + x + 2x^2$

(b) $(\sin x + x^{-2})(\cos x - x^3)$

(c) $\frac{x^2 + x + 1}{\sqrt{x} + 1}$

2. Sketch a graph and find the domain and range of $\frac{1}{\sqrt{x^4 - 2x^2} - \sqrt{3}}$

3. Show that $\frac{\sin x}{\cos^2 x} = 1$ has at least one solution in the interval $[\pi/6, \pi/4]$

4. Find a value of N such that

$$x > N \Rightarrow \left| \frac{x^2 + 20}{x^2 + 19} - 1 \right| < \frac{1}{100}$$

5. Find the equation of the line tangent to $y = \frac{x}{2} + \frac{1}{x^2} - 1$ at $x = 1$

6. Find $\lim_{x \rightarrow \infty} \sqrt{x^4 - x^2} - x^2$

7. Prove that $\lim_{n \rightarrow \infty} \frac{x+1}{2x} = \frac{1}{2}$