

Qualifying Exam Syllabus

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1 Numerical Solutions of Differential Equations

Ordinary Differential Equations:

- **Basic notions:** Convergence, consistency, A -, $A(\alpha)$ -, B - and L -stability, region of absolute stability, local truncation error, implicit and explicit schemes.
- **Runge-Kutta methods:** Order conditions, stability, collocation and implicit methods, Gaussian quadrature.
- **Multistep methods:** Order conditions, stability, collocation and implicit methods, Adam-Bashforth and Adam-Moulton, backward differentiation formula.
- **Error estimation and stepsize control:** Embedded Runge-Kutta, paired Adam-Bashforth/Adam-Moulton methods.

Partial Differential Equations:

- **Finite differences:** Stability, consistency, convergence, local truncation error, CFL condition, von Neumann stability analysis; basic schemes (leapfrog, Crank-Nicholson, Lax-Friedrichs, Lax-Wendroff, upwinding).
- **Finite elements:** Definition, Galerkin methods, weak formulation, existence of solutions, error analysis.
- **Mesheres:** Definition, Delaunay triangulation and refinement, advancing fronts, structured mesh generation.

References: (i) Randall LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations, Steady State and Time Dependent Problems*, and (ii) Arie Iserles, *A First Course in the Numerical Analysis of Differential Equations*, 2nd edition.

2 Computational Biology

- **Graphical and statistical models:** Linear models, toric models, Birch's theorem, Markov models, hidden Markov models, implicitization, directed and undirected Hammersley-Clifford.
- **Inference:** EM algorithm, Baum-Welsh algorithm, maximum likelihood estimation, tropical arithmetic, MAP inference, tree-joining algorithm, iterative proportional fitting.
- **Trees:** Phylogenetic trees, neighbour-joining algorithm, splits equivalence theorem, four-point condition, cherry-picking theorem.
- **Sequence alignment:** Needleman-Wunsch algorithm.
- **Evolutionary models:** Statistical models for mutation, Jukes-Cantor model.

Reference: Lior Pachter & Bernd Sturmfels, *Algebraic Statistics for Computational Biology*, Chap. 1-4.

3 Minor Topic: Functional Analysis

- **Banach spaces:** Open mapping theorem, closed graph theorem, uniform boundedness principle, dual spaces, Hahn-Banach theorem, weak and weak-* topologies, Alaoglu's theorem.
- **Hilbert spaces:** Inner products, Cauchy-Schwarz Inequality, orthonormal bases, weak topology.
- **Riesz representation theorems:** Dual of a Hilbert space, positive linear functionals on C_c , dual of C_0 .
- **Operators on Hilbert spaces:** Polar decomposition, compact operators, Fredholm theory, trace-class operators, Hilbert-Schmidt operators, spectral theorem.
- **Examples:** Function spaces L^p , C , C_c , C_0 ; sequence spaces ℓ^p , c , c_0 .

References: (i) Gerald Folland, *Real Analysis: Modern Techniques and Their Applications*, Chap. 5-7, and (ii) Michael Reed & Barry Simon, *Functional Analysis: Vol. 1*.