Prof. Bjorn Poonen
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MATH 160 MIDTERM

Do not write your answers on this sheet. Instead please write your name, your student ID, and all your answers in your blue books. Total: 100 pts., 50 minutes.

(1) (5 pts. each) For each of (a)-(d) below: If the proposition is true, write TRUE. If the proposition is false, write FALSE. (Please do not use the abbreviations T and F, since in handwriting they are sometimes indistiguishable.) No explanations are required in this problem.

(a) There exist integers x and y satisfying $709x + 100y = 4$.

(b) Starting from any two distinct points in the plane, it is possible to construct a regular 7-gon using straightedge and compass.

(c) Starting from two points in the plane 1 unit apart, it is possible to construct a circle of radius $\sqrt{\sqrt{2} + \sqrt{3}}$ using straightedge and compass.

(d) The equation $x^2 - 9y^2 = 1$ has infinitely many integer solutions.

(2) (30 pts.) Prove that there are infinitely many primes congruent to 2 modulo 3.

(3) (30 pts.) Either find a parametrization of the rational solutions to $x^2 + 2y^2 = 7$, or prove that no rational solutions exist.

(4) (20 pts.) Show that for any nonzero real number a, the projective closure of the plane curve $y^2 = ax^3$ is projectively equivalent to the projective closure of the curve $y = x^3$.

This is the end! At this point, you may want to look over this sheet to make sure you have not omitted any problems. Check that your answers make sense! Please take this sheet with you as you leave.