Solutions might contain less detail than what you should show.

Problem 1 (5 points, from the homework) Use trignometric substitution to evaluate the integral

$$\int_0^2 x^3\sqrt{x^2+4}\,dx$$

Solution: *We substitute* $x = 2\tan\theta$. *Then* $dx = 2\sec^2\theta\,d\theta$ *and our limits of integration become* 0 *and* $\pi/4$. *We arrive at the integral*

$$32\int_0^{\pi/4} \sec^3\theta\tan^3\theta\ d\theta \ = \ 32\int_0^{\pi/4} \sec^2\theta(\sec^2\theta\ -\ 1)\sec\theta\tan\theta\ d\theta$$

(We have used the fact that $\tan^2\theta = \sec^2\theta - 1$.*) We solve this integral by substituting* $y = \sec\theta$, *and we get*

$$\left.\frac{32\sec^5\theta}{5} - \frac{32\sec^3\theta}{3}\right]_0^{\pi/4}$$

Problem 2 (3 points)

Part a. Can x^2+3x+3 be factored, without using complex numbers? Why or why not?

Solution: *No, because the discriminant is* $-3 < 0$

Part b. (You do not need to solve for any unknown coefficients.) Suppose $p(x)$ is a polynomial of degree 6. Rewrite the following rational function as a sum of rational functions, each of whose denominators is a power of an irreducible polynomial. You may use undetermined coefficients $A, B, C, D, E, F, G, H, I, J$.

$$\frac{p(x)}{(1+x)(1+x^2)^2(2x-1)^3(x^2+3x+3)}$$

Solution:

$$= \frac{A}{1+x} + \frac{Bx+C}{1+x^2} + \frac{Dx+E}{(1+x^2)^2} + \frac{F}{2x-1} + \frac{G}{(2x-1)^2} + \frac{H}{(2x-1)^3} + \frac{Ix+J}{x^2+3x+3}$$

Problem 3 (5 points, from the homework) Evaluate

$$\int_0^1 \frac{(x-1)\,dx}{x^2+3x+2}$$

Solution: *The denominator factors as* $(x+2)(x+1)$*, so we have*

$$\frac{(x-1)\,dx}{x^2+3x+2} = \frac{A}{x+2} + \frac{B}{x+1}$$

Clearing denominators and solving for A and B gives $A = 3$ and $B = -2$. So we can rewrite our integral as

$$\int_0^1 \frac{3}{x+2} + \frac{-2}{x+1}\,dx \;=\; 3\,ln\mid x+2\mid -2\,ln\mid x+1\mid\,]_0^1$$

Problem 4 (6 points) (If you make any substitution, you do **not** have to substitute back. Just solve the new integrals you get.) For $-\pi/2 < t < 0$, evaluate

$$\int \sqrt{1-\cos^4 t}\;dt$$

You may use (without proof) that $\int \sec^3 t\,dt = \frac{1}{2}(\sec t \tan t + ln\mid \sec t + \tan t\mid) + C$

Solution: *Note that* $1-\cos^4 t = \sin^2 t(1+\cos^2 t)$*. So we can rewrite our integral as*

$$\int -\sin t\sqrt{1+\cos^2 t}\;dt$$

The minus sign on $\sin t$ *comes from the fact that* $\sin t < 0$ *on the specified range of* t *in the problem. We now substitute* $\cos t = \tan u$ *and arrive at the integral*

$$\int \sec^3 u\,du$$

whose antiderivative is given to us in the problem statement.

Problem 5 (6 points) Make a substitution into the following integral to rewrite it as the integral of a rational function. You may leave the denominator factorized if you wish, and you do not have to solve the resulting integral.

$$\int \frac{(\tan t)^{1/3}\, dt}{(\tan t)^{1/5} + (\tan t)^{1/2}}$$

Solution: *The desired substitution is* $u^{30} = \tan t$. *Then*

$$30u^{29}\, du = \sec^2 t\, dt = (1 + \tan^2 t)\, dt = (1 + u^{60})\, dt$$

and hence we obtain the integral

$$\int \frac{30u^{29}u^{10}\, du}{(u^6 + u^{15})(1 + u^{60})}$$

which is the integral of a rational function.