PROBLEM 4.4.77

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Problem:

If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is:

$$A = A_0 \left(1 + \frac{r}{n}\right)^{nt}$$

If we let $n \to \infty$, we refer to the **continuous compounding of interest**. Use l'Hopital's rule to show that if interest is compounded continuously, then the amount after t years is:

$$A = A_0 e^{rt}$$

Solution: All you gotta do is show that:

$$\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{nt} = e^{rt}$$

1) Let $y = \left(1 + \frac{r}{n}\right)^{nt}$
2) $\ln(y) = nt \ln(1 + \frac{r}{n})$
3) The important thing to realize is that you're taking the limit as n goes to ∞, which means that r and t are **constants**!

$$\lim_{n\to\infty} \ln(y) = \lim_{n\to\infty} nt \ln(1+\frac{r}{n}) = \lim_{n\to\infty} \frac{\ln(1 + \frac{r}{n})}{\frac{1}{nt}} = \lim_{n\to\infty} \frac{\frac{\frac{-r}{n^2}}{1+\frac{r}{n}}}{-\frac{1}{n^2 t}} = \lim_{n\to\infty} \frac{\frac{rn^2 t}{n^2}}{1 + \frac{r}{n}} = \lim_{n\to\infty} \frac{rt}{1 + \frac{r}{n}} = \frac{rt}{1+0} = rt$$

4) So $\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{nt} = e^{rt}$, and hence $\lim_{n\to\infty} A_0 \left(1 + \frac{r}{n}\right)^{nt} = A_0 e^{rt}$

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