PROBLEM 2.3.64

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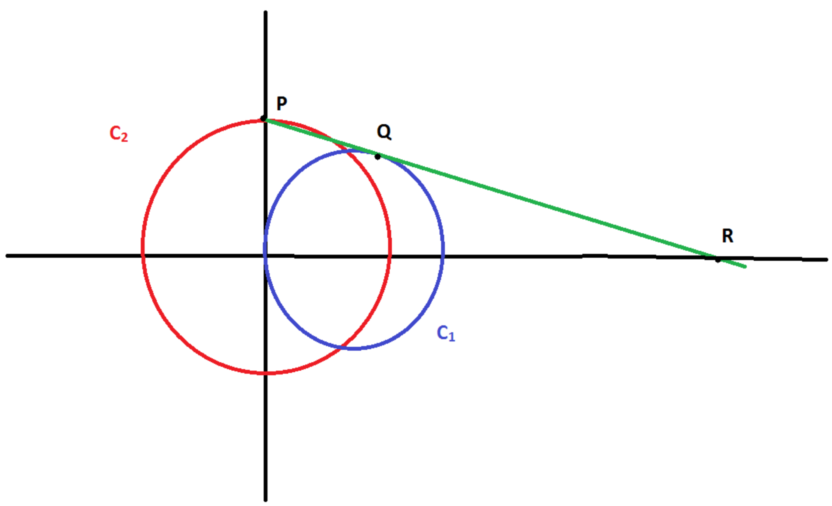
Problem: The figure shows a fixed circle C_1 with equation $(x-1)^2+y^2 = 1$ and a shrinking circle C_2 with radius r and center the origin. P is the point $(0, r)$, Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the $x-$axis. What happens to R as C_2 shrinks, that is, as $r \to 0^+$?

Picture:



Hints: Use the following steps:

(a) Find the coordinates of Q. For this, solve for x and y in the system of equations:

$$\begin{cases} (x-1)^2 + y^2 = 1 \\ x^2 + y^2 = r^2 \end{cases}$$

For this, plug in $y^2 = r^2 - x^2$ in the first equation and solve for x, then solve for y in $y^2 = r^2 - x^2$; remember that you want $x > 0$ and $y > 0$, according to the picture). The answer gives you the coordinates of Q

(b) Now that you know the coordinates of P and Q, find the equation of the line going through P and Q

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(c) Find the x-intercept of that line (set $y = 0$ and solve for x)
(d) Finally, take the limit as $r \to 0^+$ of the answer you found in (c). To do this, multiply as usual by the conjugate form.

Answers:

(a) $Q = (\frac{r^2}{2}, r\sqrt{1 - \frac{r^2}{4}})$
(b) $y = \frac{2}{r}\left(\sqrt{1 - \frac{r^2}{4}} - 1\right)x + r$
(c) $x-$ intercept $= \frac{r^2}{2\left(1-\sqrt{1-\frac{r^2}{4}}\right)}$
(d) $\boxed{4}$