Optimization

Peyam Ryan Tabrizian

Monday, November 4th, 2013

How to solve optimization problems

1) **Draw a picture!**, labeling all your variables. This time, you **can** put numbers on your picture (in sharp contrast to related rates problems!)

2) Find a function of **one variable** which you want to maximize/minimize (here is where you use *all* the info that is given to you)

3) Find the constraint. Sometimes, there's an 'open interval' constraint like $x > 0$, and sometimes there's a 'closed interval' constraint, like $3 \leq x \leq 6$)

4) Find the absolute maximum or minimum of your function in 2) given the constraint in 3). If your constraint is a **closed** interval, use the *closed interval method* from section 4.1. In all the other cases, just solve for $f'(x) = 0$ and write 'by FDTAEV'.

Note: Sometimes, it's useful to maximize the *square* of your function instead of your function (in order to avoid square roots)

List of tricks (unlike for related rates, solving an optimization problem relies more on ingenuity than on memorizing formulas)

– Formula for the distance between two points (x, y) and (x_0, y_0):

$$D = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

– Formulas for areas and/or volumes

- Volume of a cone: $V = \frac{\pi}{3} r^2 h$
- Volume of a cylinder: $V = \pi r^2 h$
- Surface area of a cylinder (w/o top and bottom): $S = 2\pi r h$

Note: FDTAEV [First Derivative Test for Absolute Extreme Values]

If $f'(c) = 0$, f is decreasing to the left of c and f is increasing to the right of c, then $f(c)$ is the absolute minimum of f (and vice-versa for absolute maximum)

Problem 1

Find the dimensions of the rectangle with perimeter 16 whose area is as large as possible.

Problem 2

Find the point on the line $2x + y = 1$ that is closest to the point $(-3, 1)$

Problem 3

(a) A box with an open top and a square base is to be made from $300cm^2$ of cardboard. What is the largest possible volume of such a box?

(b) A box with an open top and a square base is to have volume $4000cm^3$. What is the minimum amount of cardboard necessary to construct the box?

Problem 4

Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

Problem 5

[HARD; if time permits] A cone-shaped drinking cup is made frm a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.

1A/Practice Exams/Drinking cup.png