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Supersymmetries and supercharges.

N is a spacetime (a supermanifold) (a Riemannian or Lorentzian manifold). F is a superspace of fields. Lagrangian $L \in \Omega^{0,n}(F \times N)$.

We have classical solutions $M \subset F$.

Variational 1-forms. $\gamma \in \Omega^{1,n-1}_{\mathrm{loc}}(F \times N)$.

Pick γ such that $E(L) = \delta L + d\gamma \in \Omega^{1,n}_{\mathrm{loc}}(F \times M)$ does not contain variations of derivatives of fields, just variations of the fields themselves.

Remarks: (1) If L only depends on the 1-jet of fields, γ is uniquely determined. (2) By Taken's theorem γ is generally only determined up to a d-closed (d-exact) form. (3) For the usual mechanics the restriction of γ to M is $\sum_i p_i dq^i$.

Examples: (1) Classical particles on a Riemannian manifold M. $N = \mathbf{R}$, $F = \mathrm{Map}(\mathbf{R}, M)$, $L = (1/2)|\dot{x}|^2 dt$.

$\delta L = \langle \delta_\nabla \dot{x}, \dot{x}\rangle dt = \langle \delta_\nabla dx, \dot{x}\rangle = -\langle d\delta_\nabla x, \dot{x}\rangle = -d\langle \delta x, \dot{x}\rangle + \langle \delta x, d_\nabla \dot{x}\rangle = \gamma + E(L)$, $E(L) = \langle \delta x, \nabla_{\dot{x}}\dot{x}$. Here $\dot{x}dt = dx$.

Superparticle. $N = \mathbf{R}$, $F = \{x\colon \mathbf{R} \to M, \psi \in \Gamma(x^*\pi TM)\}$, $L = (1/2)(|\dot{x}|^2 + \langle \psi, \nabla_{\dot{x}}\psi\rangle)dt$. $\delta L = -d\langle \psi, \delta_\nabla \psi\rangle + \langle \delta_\nabla \psi, d_\nabla \psi\rangle + \langle \delta x, R(\psi,\psi)dx\rangle - d\langle \delta x, \dot{x}\rangle + \langle \delta x, \nabla \dot{x}, \dot{x}\rangle$.

Equations of motion: $\nabla_{\dot{x}}\dot{x} = (1/2)R(\psi,\psi)\dot{x}$, $\nabla_{\dot{x}}\psi = 0$. $\gamma_{\mathrm{sp}} = \langle \dot{x}, \delta x\rangle + (1/2)\langle \psi, \delta_\nabla \psi\rangle$.

Symplectic structures on the space of classical solutions M. $\omega = \delta\gamma \in \Omega^{2,n-1}_{\mathrm{loc}}(F \times N)$. $\omega = (d+\delta)\mathcal{L}$ on $M \times N$, $\mathcal{L} = L + \gamma$. Hence $(d+\delta)\omega = 0$ on $M \times N$.

Classical mechanics: $d\omega = 0$ on M, hence $\omega(t) = \omega$. In general, $\Omega(t) := \int_{\{t\}\times S} \omega$. By Stokes this does not depend on t.