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Charged spinors.

Definition. A spinor is a field for a spin structure SM on M of matter type (V, ρ, h, f), where (V, ρ) is a representation of $\mathrm{Cl}(p, q)$ on $\mathbf{C}l(p, q)$, $h\colon V \times V \to \mathbf{R}$ is a bilinear form, $f\colon V \to \mathbf{R}$ is ρ-invariant. A field is a section $\psi\colon M \to SM \times_\rho V = \Sigma M$ of the spinor bundle. We have an action functional $S(\psi) = \int_M D\psi \wedge_h *\psi + *(f \circ \psi)$. Equation of motion is $D\psi + im\psi = 0$ (the Dirac equation). (Here $f(v) = -mh(v, v)$.)

We want to couple this stuff to electromagnetic field.

Bundle splicing: If G_k is a Lie group and P_k is a principal G_k-bundle over Π, then $P_1 \circ P_2 = P_1 \times_M P_2$ is a principal $G_1 \times G_2$-bundle over M. If we have connections ω_k on P_k, then $\omega_1 \circ \omega_2 = p_1^* \omega_1 \oplus p_2^* \omega_2 \in \Omega^1(P_1 \circ P_2, g_1 \oplus g_2)$. If ρ_k is a representation of G_k such that $\rho_1(g_1) \circ \rho_2(g_2) = \rho_2(g_2) \circ \rho_1(g_1)$, then $\rho_1 \times \rho_2$ is a representation of $G_1 \times G_2$.

Setup for charged spinors. We have a spin structure SM on M with Levi-Civita connection, Yang-Mills theory (G, P) and connection ω on P. We have a representation $\rho_{SM}\colon \mathrm{Spin}(p, q) \to \mathrm{GL}(V)$ and $\rho_P\colon G \to \mathrm{GL}(V)$, a bundle $\Sigma P = (SM \circ P) \times_{\rho_{SM} \times \rho_P} V$ and a Dirac operator on this bundle $D^\omega\colon \Gamma(M, \Sigma P) \to \Gamma(M, \Sigma P)$.

Definition: A charged spinor is a field for $SM \circ P$ of matter type $(V, \rho_{SM} \times \rho_P, h, f)$. Action functional is $S(\psi, \omega) = \int_M D^\omega(\psi) \wedge *\psi + *(f \circ \psi) + S_{\mathrm{YM}}(\omega)$. Dirav equation $D^\omega \psi + im\psi = 0$ and $D^\omega * F_\omega = J(\psi)$.

Electrons: $V = \Sigma = \Sigma^+ \oplus \Sigma^-$ and $\rho_P(z)(v) = z^n \cdot v$, where n is an integer number (charge). Dirac equation is $D^\omega \psi + im\psi = 0$.