Nate Watson. Yang-Mills theories.

Last time: Electromagnetic field comes from the curvature of a connection D on a U(1)-bundle.

Idea: Bohm-Aharonov effect comes from $\exp(-(i/\hbar)qS(\gamma)$.

Electromagnetic field assigns phase to a path in spacetime like a connection on a trivial bundle.

The interference depends on holonomy along the difference of paths.

If B is a disk in M we have $\exp(i\int_B F) = D(\partial B)$. Action $\int_M F\wedge *F$ gives Maxwell's equation $d*F = 0$. $dF = 0$ is automatic.

Recall the character diagram from the last talk.

Holonomies are maps $LM \to \mathrm{U}(1)$ that are holonomies of some connection on a U(1)-bundle.

Theorem: $f\colon LM \to \mathrm{U}(1)$ is a holonomy if there exists a 2-form F such that for any disk B we have $\exp(i\int_B F) = f(\partial B)$.

Charge quantization. Given an electromagnetic field F with some magnetic charge at 0 we expect flux $\int_{S_2} B\cdot d\eta = \int_{S_2} F$, which is impossible for contractible manifold because F is closed. Thus the theory excludes magnetic charges for contractible manifolds. But for non-contractible manifolds we can have magnetic charges. Now $\int_{S_2} F$ can be nonzero and sees nontriviality of the bundle and is a multiple of 2π. We set $\hbar/q = 1$ so that $m = N\cdot 2\pi\cdot\hbar/q$ hence magnetic charge is quantized in terms of any electric charge and vice versa.

Now we want to replace U(1) by a non-abelian Lie group G. Take an associated bundle (need representation of G on a generic fiber). We have a notion of a G-connection on the associated bundle $E \to M$. The curvature of this connection is an $\mathrm{End}(E)$-valued 2-form with values in g (the Lie algebra of G). Curvature is not gauge invariant. We need a number, but $F\wedge *F$ is an $\mathrm{End}(E)$-valued n-form. We should apply the trace. Yang-Mills action is $\int_M \mathrm{tr}(F\wedge *F)$. This action is gauge invariant!