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Maxwell's equations and classical electromagnetism.

Fields: E, B : time dependent vector fields on \mathbf{R}^3 . $\operatorname{div} E = \rho/\epsilon_0$, $\operatorname{curl} E = -\partial B/\partial t$, $\operatorname{div} B = 0$, $c^2 \operatorname{curl} B = j/\epsilon_0 + \partial E/\partial t$. ρ is "charge density" (coulombs per cubic meter), j is "current" (coulombs per second per square meter).

Gauß' law is an application of Stokes' theorem: $q/\epsilon_0 = \int_U \rho/\epsilon_0 = \int_U \operatorname{div} E = \int_{\partial U} E \cdot n$.

Ampere's law: $-\int_D \partial B/\partial t = -d\Phi_B/dt = \int_D \operatorname{curl} E = \int_{\partial D} E \cdot ds$.

No magnetic charges (monopoles): $\operatorname{curl} B = 0$. If $\rho = j = 0$ then we get a symmetry between E and B .

Charge conservation: $\operatorname{div} j/\epsilon_0 = -\operatorname{div} (\partial E/\partial t) = -(\partial/\partial t) \operatorname{div} E = -(\partial/\partial t) \rho/\epsilon_0$, hence $\operatorname{div} j = -\partial \rho/\partial t$.

Static examples: $\partial B/\partial t = \partial E/\partial t = 0$.

Point charge: If $\rho = \delta(x)$ and $j = 0$, then $E = \pm(4\pi\epsilon_0)^{-1} r/|r|^3$ and $B = 0$.

Wave solutions: $\rho = j = 0$. $E = (0, E_y, E_z)$, $B = (0, B_y, B_z)$. $E_y = f(x - ct) + g(x + ct)$, $E_z = F(x - ct) + G(x + ct)$, $cB_z = f(x - ct) - g(x + ct)$, $cB_y = -F(x - ct) + G(x + ct)$. Wave with speed of propagation c .

Solenoid solution: $E = 0$ outside some cylinder, $\rho = 0$ everywhere, B is pointing upwards, j goes around the cylinder.

$\int B \cdot ds = |B|h = j/(\epsilon_0 c^2) = jnh/(\epsilon_0 c^2)$. $|B| = jn/(\epsilon_0 c^2)$. n is the number of turns per length.