Spring 2011, Math 276: Index Theory, Homework 3

Please submit by whatever date you deem appropriate.

Problem 7: The Euler class.

(a) Suppose E is an oriented real vector bundle over a closed oriented manifold M and s is a section of this bundle that is transversal to the zero section. Prove that the Poincaré dual of the manifold of zeroes of s is equal to $e(E)$. Discuss the statement in the absence of orientations.

(b) Find an oriented vector bundle with vanishing Euler class but without a non-vanishing section.

Problem 8: Topological index on odd-dimensional manifolds. Prove that the topological index of any elliptic differential operator on an odd-dimensional manifold M is zero. Hint: Consider the involution on the total space of T^*M given by negation.