

Math 54 - Midterm #2

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Name: _____

1. (10 points) Find bases for the row space and column space of the following matrix A , and determine $\text{rank}(A)$ and $\dim NS(A)$.

$$A = \begin{pmatrix} 1 & 2 & 0 & 3 & 5 \\ -2 & -1 & -2 & -7 & -11 \\ 2 & -2 & 4 & 12 & 14 \\ 4 & 11 & -2 & 7 & 17 \end{pmatrix}$$

2. (15 points) Determine the matrix that induces the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ which rotates the xz -plane by $\frac{\pi}{6}$ radians and reflects the y -axis.

3. Let A be the following symmetric matrix.

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

- (a) (10 points) Show that the eigenvalues of A are 2, 2, and 5.
- (b) (10 points) Find bases for each eigenspace of A .
- (c) (10 points) Apply the Gram-Schmidt process to the eigenvectors you found in (b).
- (d) (5 points) The resulting vectors in (c) form an orthonormal basis for \mathbb{R}^3 . Compute the coordinate vector of $(1, 0, 1)$ relative to this basis.

4. (15 points) Prove that λ is an eigenvalue of a matrix A if and only if $\det(\lambda I - A) = 0$.

5. (a) (10 points) Show that $e^x \cos x$ and $e^x \sin x$ are solutions of the differential equation

$$y'' - 2y' + 2y = 0.$$

- (b) (10 points) Show that $e^x \cos x$ and $e^x \sin x$ are linearly independent.
(c) (5 points) What can you conclude about the general solution of the above differential equation?

6. (0 points) Draw a picture portraying your love/hate relationship with linear algebra.