Dr. Martin Olbermann — Math 185, Section 1
Due: Wednesday, Feb. 6, in class — Spring 2008

Homework 2

Proofs and explanations should always be written, as often as possible, using complete English sentences. You should always explain and justify each of the steps in your solution, unless otherwise noted. Write your name and "Math 185, section 1" on the top right of the first page.

1. Regular triangles with vertices D and E, respectively, are erected outwardly on the sides AB and BC of the triangle ABC. Prove that the midpoints of BD, BE and AC are vertices of a regular triangle. (Use complex numbers.)

2. Sarason, I.8, exercises 1 (a),(b),(c) and 2 (b),(c).

(In exercise 2, prove only part (b) using ϵ and δ.)

3. We consider the unit sphere S^2 in $\mathbb{R}^3$ with the metric induced from the euclidian metric in $\mathbb{R}^3$. Show that stereographic projection $f : \mathbb{C} \to S^2 \setminus \{(0,0,1)\}$ and its inverse are continous functions. Deduce that open sets of the metric space $\mathbb{C}$ correspond under stereographic projection bijectively to open sets of $S^2 \setminus \{(0,0,1)\}$.

(This is why $S^2 \setminus \{(0,0,1)\}$ is also a good picture for $\mathbb{C}$. It follows that e.g. convergence and compactness in $\mathbb{C}$ corresponds to convergence and compactness in $S^2 \setminus \{(0,0,1)\}$. Furthermore this correspondence is useful to make $\overline{\mathbb{C}}$ a metric space: one cannot extend the metric on $\mathbb{C}$ to one on $\overline{\mathbb{C}}$, but one can extend the metric on $S^2 \setminus \{(0,0,1)\}$ to one on S^2.)

4. Look up the solutions to Sarason, II.3, exercises (i)-(iii) for real functions in your Math 104 notes and check that these proofs also apply to the case of complex functions. (You do not need to write up anything in this exercise.)

5. Sarason, II.6, exercises 1 and 2.