

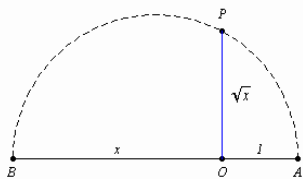
Homework 4

Proofs and explanations should always be written using complete English sentences. You should always explain and justify each of the steps in your solution, unless otherwise noted. Write your name and "Math 114" on the top right of the first page.

1. Stewart, exercise 3.9.
2. Find the degree and a basis for the field extension $\mathbb{Q}(i, \sqrt{2} + \sqrt{5}) : \mathbb{Q}$.
3. Stewart, exercise 4.8.
4. Stewart, exercise 4.12. You have to add the assumption that $\deg(p) > 1$.
5. Stewart, exercise 5.7.

Instructions:

- a) Find a formula which gives $\cos(3\phi)$ as a function of $\cos(\phi)$.
- b) Show that if $(0, 0), (1, 0) \in P_0$, then every point with coordinates in K_0 is constructible. (Show that the set of coordinates of constructible points forms a field. Use the intercept theorem to prove that it is closed under multiplication and taking multiplicative inverses.)
- c) Show that if $(0, 0), (1, 0) \in P_0$, then every point with coordinates x, y which satisfy quadratic equations with coefficients in K_0 is constructible.



- d) Apply this to $P_0 = \{(0, 0), (1, 0), (\cos \theta, \sin \theta)\}$.