

Homework 10

Proofs and explanations should always be written using complete English sentences. You should always explain and justify each of the steps in your solution, unless otherwise noted. Write your name and "Math 114" on the top right of the first page.

1. a) Show that no group of order 20 is simple. (Use the Sylow theorems to find a normal subgroup.)
b) Show that no group of order 30 is simple. (Here the Sylow theorems do not uniquely determine the number of Sylow 3-subgroups and Sylow 5-subgroups. However assume that both of these numbers are different from 1. How many elements of order 3 and of order 5 would the group have?)
2. a) Let p be a prime number. Show that every group of order p^2 is abelian. (Use the fact that every finite p -group has a non-trivial center.)
b) Let p and q be distinct prime numbers. Show that every group of order pq is soluble. (Find a normal subgroup using the Sylow theorems.)
3. Show that the Galois group of $t^p - 1$ over \mathbb{Q} is isomorphic to the group \mathbb{Z}_p^* of units of the ring \mathbb{Z}_p .
4. Stewart, exercise 14.7.
5. Stewart, exercise 15.7.