1. Let X be a metric space in which every infinite subset has a limit point. Then X is compact. To see this, note that by problem 24, X is separable, and by problem 23, X has a countable base B. To prove that X is compact, it is enough to show that every B cover has a finite subcover, by a lemma proved in class. Let C be a B cover. Since B is countable, so is C, so we can choose an enumeration $C_1, C_2, \ldots,$ of C. Let $F_i := X \setminus (C_1 \cup, \cdots C_i)$ for all i. Then $F_1, F_2, \ldots$ is a descending chain of closed sets and $\bigcap F_i = \emptyset$. We claim that some F_i is empty, meaning that some finite subset of C covers X. If not, choose some $x_i \in F_i$. If the set S of all x_i is finite, then at least one x_j belongs to infinitely many F_i, hence to all F_i, a contradiction. If the set is infinite, it has a limit point x. Every neighborhood of x contains infinitely many points of S, hence meets infinitely many of the F_i, hence meets every F_i. Thus x is in the closure of each F_i. But F_i is closed, hence x belongs to the intersection of all F_i, again a contradiction.