1. Let (X,d) be a metric space (or more generally, any Hausdorff topological space). Let A and B be disjoint compact subsets of X. Prove that there exist disjoint open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$.

2. Proof for Hausdorff spaces: Let (a,b) be a point of $A \times B$. Since X is Hausdorff, there exist disjoint neighborhoods U_{ab} of a and V_{ab} of b. Fix b, and consider $\{U_{ab} : a \in A\}$, which is an open cover of A. Hence there exists a finite number of a_i such that $\{U_{a_i,b}\}$ covers A. Let $V_b := \cap V_{a_i,b}$. Then V_b is a neighborhood of b and doesn't meet $U_b := \cup U_{a_i,b}$. Now the set of all V_b is an open cover of B, hence has a finite subcover $\{V_{b_1} \ldots V_{b_n}\}$. Each U_{b_i} is an open set containing A, and hence so is their intersection U. Since $V_{b_i} \cap U_{b_i} = \emptyset$, $V_{b_i} \cap U = \emptyset$. Hence $V := \cup V_{b_i}$ is an open neighborhood of B which doesn't meet U.