1. Prove that if F is an Archimidean ordered field containing $\mathbf{R}$, then in fact $F = \mathbf{R}$. (Hint: Suppose $x \in F$, and let $A := \{a \in \mathbf{R} : a < x\}$. Prove that A is not empty and bounded as a subset of $\mathbf{R}$.)

2. There exists a nonarchimidean field F containing $\mathbf{R}$. Here is a hint for one example, suggested by G. Bergman. Consider the set of rational functions $r(x) := f(x)/g(x)$, where $f(x)$ and $g(x)$ are polynomials with real coefficients, where $g(x)$ is not the zero polynomial, and with equality of such quotients defined in the same way as equality of rational numbers. These can be added and multiplied in the evident way, and they form a field F. This field contains the real field $\mathbf{R}$, as the constant polynomials. To define an order on F, note that $g(x)$ has only a finite number of zeroes, so $r(x)$ can be evaluated at all sufficiently large real numbers a. Then we define $r(x) < s(x)$ if and only if there exists a number N such that $s(a) > r(a)$ for all $a > N$. Prove as much as you like towards showing that this does define an ordered field. Why is it nonarchimidean?