

Let E be an uncountable subset of \mathbf{R}^n and let P the set of condensation points of E . Let \mathcal{B} be a countable base for the topology of \mathbf{R}^n , let \mathcal{B}_E denote the set of all $V \in \mathcal{B}$ such that $V \cap E$ is countable, and let $W := \bigcup \mathcal{B}_E$. Note first that if $V \in \mathcal{B}_E$, then for any $v \in V$, V is a neighborhood of v such that $V \cap E$ is countable, so v cannot be a condensation point of E . Thus $V \cap P = \emptyset$. Hence $W \cap P = \emptyset$, and hence $P \subseteq X \setminus W$. On the other hand, suppose $x \in X \setminus P$. Then x has a neighborhood which intersects E in only countably many points, and this neighborhood will contain an element of \mathcal{B} . Thus x belongs to some element of \mathcal{B}_E . In other words, $X \setminus P \subseteq W$, so $X \setminus W \subseteq P$. Taken together, these statements show that P is exactly the complement of W . Hence $E'' := (X \setminus P) \cap E = W \cap E = \bigcup (V \cap E : V \in \mathcal{B}_E)$, which is a countable union of countably many sets, hence is countable. So if $E' := E \cap P$, $E = E' \cup E''$, where E'' is countable. Now for any $x \in X$ and any neighborhood U of x , $U \cap E$ is uncountable iff $U \cap E'$ is uncountable. In particular, if p is a condensation point, then for any U , $U \cap P$ is uncountable, hence p is a limit point of P (even a condensation point).