Mathematics 254a Exercises

November 25, 2007

1. Let p be an odd prime, and let $p^* := (-1)^{\frac{p-1}{2}}$. Recall that $K := \mathbf{Q}(\sqrt{p^*})$ is the unique subfield of $\mathbf{Q}(\zeta_p)$ which has degree 2 over $\mathbf{Q}$. It follows that K is unramified over $\mathbf{Q}$ away from p. Recall that if q is an odd prime, we computed $(\frac{q}{p})$ by looking at the Frobenius element of $Gal(Kl\mathbf{Q})$ at q. Use the same method to show that

$$(\frac{2}{p}) = (-1)^{\frac{p^2-1}{8}}.$$

   Hint: Compute the integral closure of $\mathbf{Z}$ in K at 2.

2. Nuekirch, §11, number 1.

3. Nuekirch, §11, number 2

4. Nuekirch, §11, number 3