Mathematics 1A, Spring 2010 — A. Ogus
Sample Midterm Exam #2

**Instructions.**
Closed book exam — No formula sheets or notes are permitted. Calculators and other electronic devices are not allowed. Turn cell phones off and stow them in backpacks/pockets/purses.
**Show work and/or reasoning where indicated.**

**(1)** Calculate $f'(x)$, using any method from this course. Show your steps.
**(1a)** $f(x) = x^{-1}e^{2x}$
**(1b)** $f(x) = x^{\ln(x)}$
**(1c)** $f(x) = \sqrt{\arcsin(x)}$.
**(1d)** If $x^4 + xy + 2y^4 = 20$, find $dy/dx$ when $(x, y) = (2, 1)$.

**(2a)** Find the maximum value of $f(x) = x(x-1)^2$ on $[-1, 2]$, and determine all points in this interval where that value is attained.
Show all steps; you will be graded on these steps, not merely on your answer.
**(2b)** If $x$ and $y$ are positive numbers and $xy = 1$, what is the minimum possible value of $x + 2y$?
Show your steps.

**(3)** Suppose that $f$ and $f'$ are differentiable functions on an interval $(a, b)$, $c \in (a, b)$, and $f'(c) = 0$. What can one conclude if $f''(c) > 0$? What if $f''(c) = 0$? (Answer both questions.)

**(4)** Let $f(x) = x^{1/3}$. What is the equation for the linearization (also known as the linear approximation) of $f$ at 8?

**(5)** Show that if $x > 1$, then $\ln(x) < x - 1$.

# Sample problems and study guide

**Calculation problems:**
( ) Calculate $f'(x)$:
( ) $f(x) = x^3 - 8x^2 + \frac{9}{x^3}$.
( ) $f(x) = \arctan(x^2 + 1)$
( ) $f(x) = \ln(|x|)$
**Application problems:**
In general, *this exam will not stress applications at all.* (Because of time constraints; it takes longer to read and set up word problems. Expect to see these on the final exam.)
( ) Exponential growth/decay. A certain radioactive substance has a half life of $1,000$ years. If a chunk of this substance weighs 1 gram today, how much radioactive material will remain after $2,500$ years? (It is not necessary to simplify your answer in any way.)
( ) Curve sketching. This is a very important topic, but because these problems take a significant amount of time, you are not likely to be asked to draw complicated graphs on the midterm exam. You may be asked other types of questions, designed to test individual steps in the curve sketching process.
( ) Related rates problems. See the examples treated in lectures, and those assigned for homework. Many additional examples can be found in our text.
( ) Max/min problems, and optimization problems. See examples treated in lecture, and assigned problems. See text for additional problems. (These were two different sections of our text. Optimization problems are "word problems", which involve additional steps: using constraint equations to eliminate independent variables, in order to obtain a maximization or minimization problem. Solution of optimization problems sometimes requires additional skills, including curve sketching, and analysis of asymptotic behavior.)
**Calculation/theory problems:**
( ) Suppose that $f$ is a differentiable function on the interval $[0, 2]$, that $f(0) = 0$, and that $f(1) = -6$. What can we conclude about the range of the function $f'$?
( ) Show that $\sin(x) < x$ for all $x > 0$.
( ) Suppose that $f'(x) = (x + 2)(x - 1)^2(x - 3)(x - 5)$. On which intervals is $f$ an increasing function?
( ) Assuming that ln is a differentiable function, show that $\frac{d}{dx} \ln(x) = 1/x$.
**Theory questions:**
( ) State Rolle's Theorem.
( ) If $f$ has a local minimum at $c$, and if $f'(c)$ exists, show that $f'(c) = 0$.

( ) If $f'(x) > 0$ for all $x$ in an interval, then $f$ is increasing on that interval. How do we know this fact?
( ) If $f$ is a differentiable function on an interval $(a, b)$, and if $f'(x) = 0$ for every $x \in (a, b)$, then $f$ is constant. How do we know this?
( ) Let $t =$ time, and let $T(t) =$ the temperature of a liquid at time $t$. At time $t_0$, measurement reveals that $T(t_0) = 75$, $T'(t_0) < 0$, and $T''(t_0) > 0$. At a *slightly* later time $t_1 > t_0$, how would we expect $T(t_1)$ and $T'(t_1)$ to be related to $T(t_0)$ and $T'(t_0)$, respectively?
( ) Define: The graph of $f$ is concave up, over the interval $(a, b)$.
( ) Use the linearlization of a differentiable function $f$ at $a$ to estimate $f(x)$ for $x$ near $a$. Be prepared to give bounds for the error your may be making, and to predict if the estimate is too small or too large.
**(4e)** A rabbit and a hare race along a straight line, beginning at time $t = 0$. At time $t$, their positions are $r(t)$ and $h(t)$, respectively. Suppose that $r(t) = 1 + t$ for all $t$ (the rabbit is given a head start), while $h''(t) < 0$ for all $t$. What is the maximum possible number of times $t > 0$ at which $r(t) = h(t)$? Explain your answer briefly.
**(4a)** Define: $f$ has an inflection point at $x$.
**(5)** $\sec(x) = \frac{1}{\cos(x)}$, with domain $(0, \pi/2)$ in this problem. Let arcsec be the inverse function of sec. Assuming that arcsec is differentiable, show that $\text{arcsec}'(x) = \frac{1}{x\sqrt{x^2-1}}$ for $x > 0$. Show your reasoning.