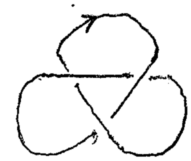
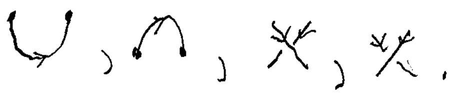
Invariants of Links

Consider planar projections of links like: 

The corresponding category is the category of tangles. E.g.

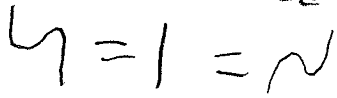
How to write this in terms of generators and relations?



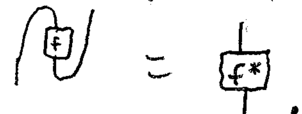
This looks familiar from 1 and 2-dimensions. All these descriptions are planar in the sense of being on a blackboard. They also have $\cap$ and $\cup$ so that pictures like

are the same.

Such a category is called Planar (or pivotal). Explicitly we need



and



Notice that if $\cup : V^* \otimes V \to \mathbb{C}$ and $\cap : \mathbb{C} \to V \otimes V^*$ then

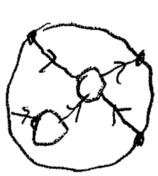
is a map $W^* \to W^*$ and

is a map $V \to V$ so it only makes sense to call them equal if we have a natural transformation $V \to V^{**}$. Algebraically these match natural conditions for a dual!!

Once we have a planar category there's no difference between inputs and outputs. So we replace all rectangles with circles:

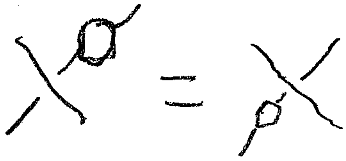
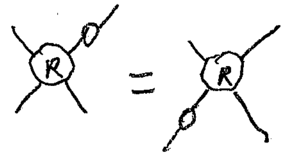
etc.

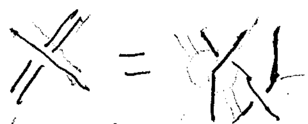
We can also do the same in the middle ~~of~~ diagrams:

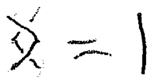


other diagrams can be put in the inner circles. Such a setup is called a planar algebra.

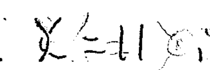
Now let's return to tangles. What are the relations for 

1)  etc. (i.e. )

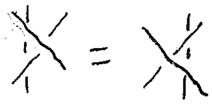
2)  etc. 4)  etc.

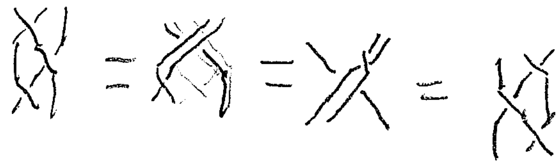
3)  etc.

Together these imply everything!

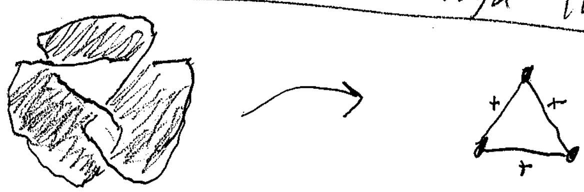
I  is 3

II  is 4

III  follows from 1) and 2).

Notice  is the braid relation.

Such a category is called a <u>braided category</u>. One often allows multiple simple objects which corresponds to a coloring of strands on the topological side. By the usual Yoga these give invariants,

Another approach  work out details

as a state sum. RIII becomes  with correct signs.

Planar algebras always contain TL