~~Topological Quantum Field Theory~~

~~Combinatorial Category Theory~~

Topological Invariants and Quantum Algebra

1. Introduction
2. 2-dim manifolds via pants
3. 2-dim manifolds via triangulations.
4. Planar Categories
5. Braided Categories
6. Category of spiders
7. 3-dim via Kirby Calculus
8. 3-dim via triangulations
9. 3-dim via Heegaard splittings
10. Khovanov Homology
11. Center construction

2-dim (2–3)

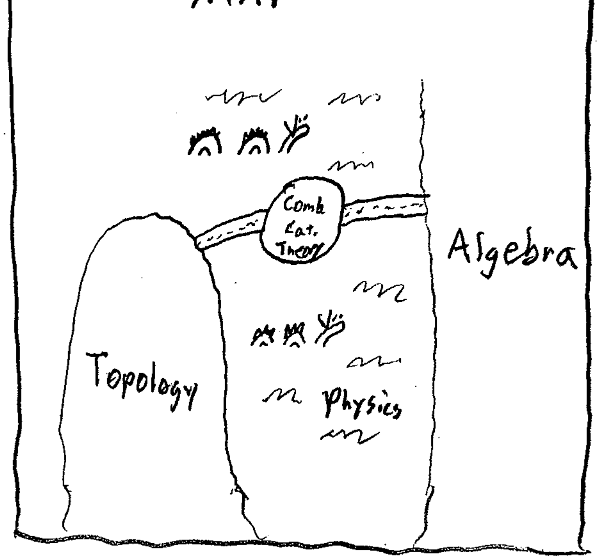
knots + links (4–6)

3-dim (7–9)

Sources

- Notes math.berkeley.edu/~nsnyder
- Frobenius Algebras + TQFTs, Kock.
- John Baez's website.
- Quantum Invariants of... , Turaev.
- Involutory Hopf algebras... , Kupperberg
- Barret + Westbury papers
- On Khovanov's Categorification... , Bar Natan
- The Center Constuction, Kassel + Turaev

MAP



We want to study invariants of some class of topolical <u>things</u>. The simplest example is compact 1-dim manifolds. Here is the recipe:

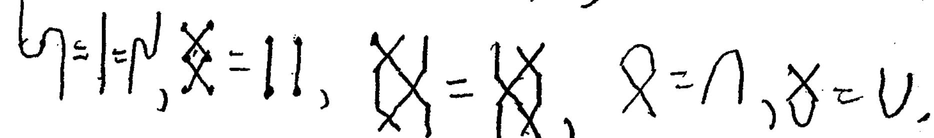
1. Instead consider <u>things</u> with boundary.
2. Turn this into a category $\mathcal{C}_{top}$ as follows:
   <u>Objects</u>: Possible boundaries
   <u>Morphisms</u>: The formal span of <u>things</u> with given boundaries

<u>Ex</u> <u>Obj</u> are disjoint unions of points. A typical basis element for $Mor(\cdots, \cdots\cdots)$ is 

3. Describe the extra structure of the category. Write down <u>explicit generators and relations</u>. The crucial observation is that $\boxed{\text{disjoint union acts like a tensor product.}}$

<u>Ex</u> Monoidal category. $\otimes$ is disj. union, $\mathbb{1}$ is the empty set. This category is <u>symmetric</u>, since $\alpha\otimes\beta \xrightarrow{\sim} \beta\otimes\alpha$, via .

It is generated by the object • and the morphisms

, with relations .

4. Call this combinatorial category $\mathcal{C}_{univ}$, it is equivalent to $\mathcal{C}_{top}$.
5. Find a particulat algebraic category $\mathcal{C}_{part}$ and a functor $F: \mathcal{C}_{univ} \to \mathcal{C}_{part}$. Distinguishing morphisms in $\mathcal{C}_{part}$ lets us distinguish morphisms of $\mathcal{C}_{univ}$, thus $\mathcal{C}_{top}$, thus <u>things</u>. Expicitly we get a topological invariant <u>things</u> $\to End(\mathbb{1})$.

In particular, suppose we have $F: \mathcal{C}_{top} \to$ <u>Vect</u>. Then we have <u>things</u> $\to End(k) = k$.

Ex. $F(\cdot) = V$. $F(\cup) : V \otimes V \to \mathbb{C}$. This form must be symmetric. Since $\cap\!\cup = |$ we need that the form is nondegenerate and $F(\cap) : \mathbb{C} \to V \otimes V$ is the associated map. So we get an invariant for any choice of vector space V with sym. bil form. One easily computes that $F(M) = (\dim V)(\dim H^0(M))$.

oops!

Exercises

1. Do the above approach for <u>oriented</u> 1-dim manifolds. There are now two basic objects • o and they go to V and V^*.
2. Allow colorings of connected componants so now we can have many basic objects. E.g. $\mathcal{C}_{part} = \underline{Rep}(G)$.