WORKSHEET #5

① Determine if the following matrices are diagonalizable:

$$\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \quad \begin{bmatrix} -5 & -2 \\ 1 & 3 \end{bmatrix}.$$

If they're, diagonalize them.

② Compute A^{10} where $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

③ Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be linear such that the standard matrix for T is $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$. Find a basis $\mathcal{B}$ of $\mathbb{R}^3$ such that $[T]_{\mathcal{B} \leftarrow \mathcal{B}}$ is diagonal.

④ Let $T: \mathbb{P}_2 \to \mathbb{P}_4$ be:

$$T(p(t)) = p(t) + t^2 p(t)$$

Find $[T]_{\mathcal{E} \leftarrow \mathcal{B}}$ where $\mathcal{B} = \{1, 1-t, 1+t^2\}$ & $\mathcal{E} = \{1, t, t^2, t^3, t^4\}$ are bases for $\mathbb{P}_2, \mathbb{P}_4$ respectively.