WORKSHEET #2

① Let $\mathbb{P}_4 = \{ p(t) \mid p$ has degree at most $4 \}$. Find a basis for $\mathbb{P}_4$ & $\dim \mathbb{P}_4$.

Solution: A basis for $\mathbb{P}_4$ is $\mathcal{B} = \{ 1, t, t^2, t^3, t^4 \}$

$\rightarrow \dim \mathbb{P}_4 = 5.$

② Let $T: V \to V$ be linear & $V = \{ f: \mathbb{R} \to \mathbb{R} \mid f$ is twice differentiable$\}$

$T(f) = \frac{df}{dx}$. Compute $\operatorname{Ker} T$ & $\dim(\operatorname{Ker} T)$.

Solution: $\operatorname{Ker}(T) = \{ f \mid \frac{df}{dx} = 0 \}$

$= \{ f \mid f$ is a constant function $\}$.

So $\operatorname{Ker} T$ is basically the same as $\mathbb{R}$ and so

$\dim \operatorname{Ker} T = \dim(\mathbb{R}) = 1.$

③ Let $T: M_{2\times 2} \longrightarrow M_{2\times 2}$ be defined as follows:

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix}$$

Compute $\operatorname{Ker} T$, $\operatorname{Range} T$, $\dim(\operatorname{Ker} T)$, $\dim(\operatorname{Range} T)$.

Solution: A basis for $M_{2\times 2}$ is $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

So $\dim(M_{2\times 2}) = 4.$

$\operatorname{Range}(T) = \left\{ \begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix} \middle| \; b, c \in \mathbb{R} \right\}$ so $\dim(\operatorname{Range} T) = 2.$

$\operatorname{Ker}(T) = \left\{ \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \middle| \; a, d \in \mathbb{R} \right\}$ so $\dim(\operatorname{Ker} T) = 2.$

(4) Is $\{\sin x, \cos x, x, \sin(x+\pi/4)\}$ linearly independent?

Solution: NO because

$$\sin(x+\pi/4) = \sin x \cos \pi/4 + \sin \pi/4 \cos x$$

$$= \frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x$$

i.e. $\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x - \sin(x+\pi/4) = 0$

So $\{\sin x, \cos x, \sin(x+\pi/4)\}$ is linearly dependent.

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