WORKSHEET # 1

① Find the matrix of the linear transformation $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ that is the composition of (first) rotation by $30°$ and (second) reflection about the origin.

② Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear transformation defined by:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ x_1 + x_3 \\ x_2 - x_3 \end{bmatrix}$$

Find the matrix of T. Is T one to one? Is T onto?

③ Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$, $\vec{v}_4 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

+ Is $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ linearly independent?
+ what is span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$.

④ Suppose A is a 3×3 matrix & $\vec{b} \in \mathbb{R}^3$ is such that $A\vec{x} = \vec{b}$ does not have solutions. Can there be a $\vec{c} \in \mathbb{R}^3$ such that $A\vec{x} = \vec{c}$ has a unique solution?