

- 1) Find the domain of $f(x) = \frac{\sqrt{x} + \sqrt{4-x}}{\sin(x)(x^2+1)}$.
- 2) Show whether the following functions are even, odd, or neither.
 $f(x) = x \cos(x + 2\pi) \sin(2x)$
 $g(x) = x|x|$
 $h(x) = x^4 + \cos(x + \pi/2)$
 $b(x) = p(x)q(x)$ where p is odd and q is even.
- 3) Find the domain of $f \circ g$ and $g \circ f$, where $f(x) = \sqrt{x} - 1$ and $g(x) = \frac{1}{x^2-2}$.
- 4) Graph the function $f(x) = -\frac{1}{2} \cos(2x - 3) + 2$.
- 5) Let f be even and g be odd. Is $h = f \circ g$ even or odd? What about $p = g \circ f$?
- 6) Prove using the definition of limit that $\lim_{x \rightarrow 3} \left(\frac{1}{3}x - 4\right) = -3$.
- 7) Prove using the definition of limit that $\lim_{x \rightarrow 2} x^2 = 4$.
- 8) Evaluate $\lim_{x \rightarrow 0} \frac{\cos(x) - \cos^2(x)}{x}$.
- 9) Evaluate $\lim_{x \rightarrow 0} x \cot(3x)$.
- 10) Evaluate $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7}$.
- 11) Evaluate $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$.
- 12) Prove that $\lim_{x \rightarrow 0^+} \sqrt{x} \left(1 + \sin^2\left(\frac{2\pi}{x}\right)\right) = 0$.
- 13) Prove that $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) = 0$.
- 14) Show that $\sin^2(x) + \cos(3x) = x^3 + 2x$ has a real solution.
- 15) The gravitational force exerted by the Earth on a unit mass at a distance r from the center of the planet is
- $$F(r) = \begin{cases} \frac{GMr}{R^3} & r < R \\ \frac{GM}{r^2} & r \geq R \end{cases}$$
- where M is the mass of the Earth, R is its radius, and G is the gravitational constant. Is F continuous function of r ?
- 16) a) Show that $F(x) = |x|$ is continuous everywhere.
b) Prove that if f is continuous on an interval then so is $|f|$.
c) Is it true that if $|f|$ is continuous then so is f ? If true, prove it. Otherwise find a counterexample.
- 17) Prove using the definition of limit that $\lim_{x \rightarrow \infty} \frac{2}{(x-2)^3} = 0$.
- 18) Prove using the definition of limit that $\lim_{x \rightarrow -\infty} \frac{2}{(x-2)^3} = 0$.
- 19) Prove using the definition of limit that $\lim_{x \rightarrow \infty} \sqrt{x^2 + 9} = \infty$.
- 20) Prove using the definition of limit that $\lim_{x \rightarrow 2} \frac{2}{(x-2)^4} = \infty$.
- 21) Prove using the definition of limit that $\lim_{x \rightarrow 3^-} \frac{1}{(2x-6)^3} = -\infty$.
- 22) Prove using the definition of limit that $\lim_{x \rightarrow 1^+} \frac{3|x-1| + 2x - 2}{x-1} = 5$.

23) Find all functions that are both even and odd.

24) Evaluate $\lim_{x \rightarrow \infty} \sqrt{16x^2 + 2x - 1} - 4x$.

25) Evaluate $\lim_{x \rightarrow \infty} \frac{4x^4 + 5}{(x^2 - 2)(2x^2 - 1)}$.

26) Show whether the following function is differentiable or not differentiable at $x = 0$.

$$f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

27) Show whether the following function is differentiable or not differentiable at $x = 0$.

$$f(x) = \begin{cases} x \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

28) Using the limit definition of derivative find $f'(x)$ where $f(x) = \frac{x^2+1}{x-2}$.

29) Using the limit definition of derivative find $f'(x)$ where $f(x) = \sqrt{2x+1}$.

30) Show whether $f(x) = |x|$ is differentiable or not differentiable at $x = 0$.

31) Show that the derivative of an even function is odd.

32) Show that the derivative of an odd function is even.

33) Find the equation for the line tangent to the graph $f(x) = \cos(\pi x) + x^2$ at $x = 1$.

34) Differentiate $f(x) = \frac{2x^3+4x^2+1}{\sqrt[3]{x}}$.

35) For what values of x does the graph of $f(x) = -x + 3\cos(x+1)$ have a horizontal tangent.

36) Find a cubic polynomial with horizontal tangents at the points $(-2, 6)$ and $(2, 0)$.

37) Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the following values:

a) $(fg)'(5)$

b) $(f/g)'(5)$

c) $(g/f)'(5)$

38) The equation for the height of a ball being thrown directly upward from a cliff is $s(t) = 16 + 12t - 5t^2$. Find the equations for the velocity and the acceleration of the ball as functions of t .

39) Differentiate $f(x) = \frac{x}{x + \frac{2}{x}}$.

40) Prove that $\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$.

41) Prove that if g is differentiable then $\frac{d}{dx} \left[\frac{1}{g(x)} \right] = \frac{-g'(x)}{[g(x)]^2}$.

42) The following figure shows the graphs of f , f' , and f'' . Identify each curve, where $a(x)$ is the solid graph, $b(x)$ is the dotted graph, and $c(x)$ is the dashed graph.

