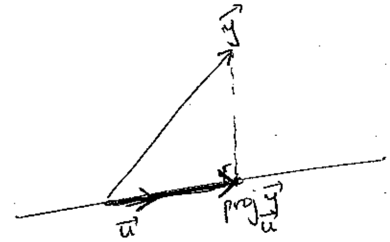
QUIZ #6

① (3 pts) Let $\vec{y} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Compute the distance from $\vec{y}$ to the line through $\vec{u}$ and the origin.



$$\text{proj}_{\vec{u}} \vec{y} = \frac{\vec{y}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\vec{u} = \frac{15}{5}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

So distance between $\vec{y}$ & the line is:

$$\text{dist}(\vec{y}, \text{proj}_{\vec{u}}\vec{y}) = \|\vec{y} - \text{proj}_{\vec{u}}\vec{y}\| = \left\|\begin{bmatrix} -3 \\ 9 \end{bmatrix} - \begin{bmatrix} 3 \\ 6 \end{bmatrix}\right\|$$

$$= \left\|\begin{bmatrix} -6 \\ 3 \end{bmatrix}\right\| = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}.$$

② (3 pts) Find a least-squares solution to $A\vec{x} = \vec{b}$ where:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}.$$

We solve: $A^T A \vec{x} = A^T \vec{b}$

$$A^T A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 24 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \end{bmatrix}$$

So $\vec{x} = \begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$.

So $\begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$ is a least-squares solution to $A\vec{x} = \vec{b}$.

③ (4 pts) Find the projection of $\vec{v}$ onto the subspace spanned by $\vec{v_1}, \vec{v_2}$

where $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ -2 \end{bmatrix}$; $\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$.

Solution: $\vec{v_1} \cdot \vec{v_2} = 1 \neq 0$ so they're not orthogonal

Let $\vec{w_1} = \vec{v_1}$

$$\vec{w_2} = \vec{v_2} - \frac{\vec{v_2} \cdot \vec{w_1}}{\vec{w_1} \cdot \vec{w_1}} \vec{w_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4/5 \\ 1 \\ 0 \\ -2/5 \end{bmatrix}$$

So $\vec{w_1} \cdot \vec{w_2} = 0$ & $\text{span}\{\vec{w_1}, \vec{w_2}\} = \text{span}\{\vec{v_1}, \vec{v_2}\}$.

So $$\text{proj}_{\text{span}\{\vec{w_1}, \vec{w_2}\}} \vec{v} = \frac{\vec{v} \cdot \vec{w_1}}{\vec{w_1} \cdot \vec{w_1}} \vec{w_1} + \frac{\vec{v} \cdot \vec{w_2}}{\vec{w_2} \cdot \vec{w_2}} \vec{w_2}$$

$$= \frac{-2}{5} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix} + \frac{17/5}{45/25} \begin{bmatrix} 4/5 \\ 1 \\ 0 \\ -2/5 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{5} \\ 0 \\ 0 \\ -\frac{4}{5} \end{bmatrix} + \frac{17}{9} \begin{bmatrix} 4/5 \\ 1 \\ 0 \\ -2/5 \end{bmatrix} = \begin{bmatrix} \frac{10}{9} \\ 17/9 \\ 0 \\ \frac{-14}{9} \end{bmatrix}$$