QUIZ #4

① a) (2 pts) Determine if the set H of matrices of the form $\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}$ is a subspace of $M_{2\times 2}$.

b) (2 pts) Is $\{\sin x, \cos x, x\}$ linearly independent? why?

1a) + $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$: the zero vector of $M_{2\times 2}$ is in H (just take $a=c=d=0$)

+ Let $A = \begin{pmatrix} a_1 & 0 \\ c_1 & d_1 \end{pmatrix}$ $B = \begin{pmatrix} a_2 & 0 \\ c_2 & d_2 \end{pmatrix}$ be two vectors in H

then $A + B = \begin{pmatrix} a_1+a_2 & 0 \\ c_1+c_2 & d_1+d_2 \end{pmatrix}$ is also in H.

+ Let $A = \begin{pmatrix} a & 0 \\ c & d \end{pmatrix}$ be in H, $e \in \mathbb{R}$ then

$eA = \begin{pmatrix} ea & 0 \\ ec & ed \end{pmatrix}$ is in H.

$\therefore$ $H \trianglelefteq M_{2\times 2}$.

1b) YES because if $a\sin x + b\cos x + cx = 0$

then

for $x = 0$: $b = 0$

$$\left.\begin{array}{ll} x = \frac{\pi}{2}: & a + c\frac{\pi}{2} = 0 \\ x = 1: & a\sin 1 + c = 0 \end{array}\right\} \longrightarrow a = c = 0.$$

② (3 pts) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} -7 \\ 9 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \end{bmatrix} \right\}$ be two distinct bases for $\mathbb{R}^2$. Find $\begin{bmatrix} -7 \\ 9 \end{bmatrix}_{\mathcal{B}}$, $\begin{bmatrix} -5 \\ 7 \end{bmatrix}_{\mathcal{B}}$, and $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$.

$$\begin{bmatrix} -7 \\ 9 \end{bmatrix} = a\begin{bmatrix} 1 \\ -3 \end{bmatrix} + b\begin{bmatrix} -2 \\ 4 \end{bmatrix} \rightarrow a = 5, b = 6$$

so $\begin{bmatrix} -7 \\ 9 \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} -5 \\ 7 \end{bmatrix} = a\begin{bmatrix} 1 \\ -3 \end{bmatrix} + b\begin{bmatrix} -2 \\ 4 \end{bmatrix} \rightarrow a = 3, b = 4$$

So $\begin{bmatrix} -5 \\ 7 \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

So $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = \begin{bmatrix} 5 & 3 \\ 6 & 4 \end{bmatrix} \rightarrow \underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \underset{\mathcal{B}\leftarrow\mathcal{C}}{P}^{-1} = \cancel{\begin{bmatrix} 2 & -3 \\ -\frac{3}{2} & \frac{5}{2} \end{bmatrix}}$

$$= \begin{bmatrix} 5 & 3 \\ 6 & 4 \end{bmatrix}^{-1} = \frac{1}{20-18}\begin{bmatrix} 4 & -3 \\ -6 & 5 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -3 \\ -6 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3/2 \\ -3 & 5/2 \end{bmatrix}$$

③ (3 pts) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & -3 \end{bmatrix}$. Find a Row(A)'s basis, and compute dim(Col A).

$$A \xrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -2 & -3 \\ 0 & -3 & -6 & -11 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - \frac{3}{2}R_2} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -2 & -3 \\ 0 & 0 & -3 & -\frac{13}{2} \end{bmatrix}$$

So a basis for Row A is:

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -3 \\ -\frac{13}{2} \end{bmatrix} \right\}$$

$\rightarrow$ dim Col A = dim Row A = 3.