QUIZ # 3

① a) (2 pts) Let A, P be square matrices of the same size. Show that $\det(P^{-1}AP) = \det A$.

b) (2 pts) True / False:

"The determinant of A is the product of diagonal entries of A."

Explain your answer.

a) $\det(P^{-1}AP) = \det(P^{-1})\det(A)\det(P)$

$= \det(P^{-1})\det P \det A$

$= \det(P^{-1}P)\det A$

$= \det(I)\det A = \det A.$

b) FALSE

<u>Counterexample:</u> Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ $\det(A) = 1.1 - 2.3 = -5 \neq 1.1.$

② (3 pts) Compute $\det A$ where $A = \begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & 9 \\ -1 & 7 & 0 \end{bmatrix}$

Expand along the 3rd row:

$$\det A = (-1)\det\begin{bmatrix} -4 & 2 \\ 8 & 9 \end{bmatrix} - 7\det\begin{bmatrix} 1 & 2 \\ -2 & 9 \end{bmatrix}$$

$$= (-1)(-36-16) - 7(9+4)$$

$$= 52 - 91 = -39.$$

③ (3 pts) Find rank A, Nullity A where $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 + 3R_1} \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & +5 & 10 & -10 \\ 0 & 0 & 1 & 2 & -2 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 5 & 10 & -10 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Columns 1, 3 have a pivot.
The others don't

So a basis for Col(A) is $\left\{ \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} \right\} \longrightarrow \text{Rank}(A) = 2$

Since $\text{Rank}(A) + \text{Nullity}(A) = 5$, $\text{Nullity}(A) = 3$.