QUIZ #2

① a) (2pts) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & -2 \end{bmatrix}$. Determine if A is invertible.

If A is invertible, find A^{-1} & check your answer.

b) (2pts) Write the solution set of the following system in parametric vector form:

$$\begin{cases} x_1 + 3x_2 - 5x_3 = 0 \\ x_1 + 4x_2 - 8x_3 = 0 \\ -3x_1 - 7x_2 + 9x_3 = 0 \end{cases}$$

a) $$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & -1 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -2 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & -2 & 1 & 1 \end{array}\right] \xrightarrow{R_3 \leftarrow -\frac{R_3}{2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

So A is invertible and $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

It's easily seen that $AA^{-1} = A^{-1}A = I$

b) $$\begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 + 3R_1]{R_2 \leftarrow R_2 - R_1} \begin{bmatrix} 1 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 2 & -6 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - 2R_2} \begin{bmatrix} 1 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

So x_3 is free $x_2 = 3x_3$, $x_1 = -3x_2 + 5x_3 = -4x_3$

→ Solutions of the system are of the form:

$$\begin{bmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

② (3 pts) Find all values of h such that the vectors

$$\vec{v_1} = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix} \quad \vec{v_2} = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix} \quad \vec{v_3} = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}$$

are linearly independent.

Suppose $a\vec{v_1} + b\vec{v_2} + c\vec{v_3} = \vec{0}$

i.e. $\begin{bmatrix} a \\ -5a \\ -3a \end{bmatrix} + \begin{bmatrix} -2b \\ 10b \\ 6b \end{bmatrix} + \begin{bmatrix} 2c \\ -9c \\ hc \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Then $\begin{cases} a - 2b + 2c = 0 \\ -5a + 10b - 9c = 0 \\ -3a + 6b + hc = 0 \end{cases}$

The corresponding matrix is $\begin{bmatrix} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & h \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & h-6 \end{bmatrix}$

Since the matrix does not have a pivot in every column, $\vec{v_1}, \vec{v_2}, \vec{v_3}$ are linearly dependent. So no h can make them linearly independent.

(Another way to see this is: $\{\vec{v_1}, \vec{v_2}\}$ is linearly dependent, so $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ is linearly dependent no matter what $\vec{v_3}$ is).

③ (3 pts) Find the standard matrix of the linear transformation $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ that first rotates points through $-\frac{\pi}{4}$ (rad) and then reflects points through the y-axis.

$T(\vec{e_1}) = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$

$T(\vec{e_2}) = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$

So $A = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$ is the standard matrix for T.