QUIZ # 1

1) (4 pts)

a) Find the general solutions of the system whose <u>augmented matrix</u> is:

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 3 \\ 2 & 1 & 2 & -2 \end{array}\right]$$

b) Given is the following system:

$$\begin{aligned} 4x_1 + x_2 + 3x_3 &= 9 \\ x_1 - 7x_2 - 2x_3 &= 2 \\ 8x_1 + 6x_2 - 5x_3 &= 15 \end{aligned}$$

Turn this system into an equation $A\vec{x} = \vec{b}$ an an equation $x_1\vec{a_1} + x_2\vec{a_2} + x_3\vec{a_3} = \vec{b}$.

a) (2pts)

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 3 \\ 2 & 1 & 2 & -2 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 4 & 1 & 3 \\ 0 & -7 & 0 & -8 \end{array}\right] \quad 1pt$$

$-7x_2 = -8 \rightarrow x_2 = \frac{8}{7}$ $\quad x_3$: free

$x_1 = -4x_2 - x_3 + 3 = -\frac{32}{7} - x_3 + 3 = \frac{-11}{7} - x_3$

The general solution is:

$$\begin{bmatrix} -\frac{11}{7} - x_3 \\ 8/7 \\ x_3 \end{bmatrix}, \quad x_3 : \text{any real number.}$$

1 pt

b) (2pts)

$$\underbrace{\begin{bmatrix} 4 & 1 & 3 \\ 1 & -7 & -2 \\ 8 & 6 & -5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 9 \\ 2 \\ 15 \end{bmatrix}}_{\vec{b}} \quad (1pt)$$

$$\vec{a_1} = \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} \quad \vec{a_2} = \begin{bmatrix} 1 \\ -7 \\ 6 \end{bmatrix} \quad \vec{a_3} = \begin{bmatrix} 3 \\ -2 \\ -5 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -7 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 15 \end{bmatrix}$$

(1pt)

② (3 pts) Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$.

Is $\vec{b}$ in span{columns of A} ? Ans: Yes ...

If $\vec{b}$ in span{columns of A}, that means $\vec{b}$ can be represented by linear combinations of the column vectors in A.

So $\Rightarrow$ $\vec{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$.

$\Rightarrow \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$.

$\Rightarrow$ Solve for arguement matrix.

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 \times \frac{1}{2} \rightarrow R_1 \\ R_2 + R_3 \rightarrow R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 6 & 6 & 6 \\ 1 & -2 & 1 & 3 \end{array}\right] \begin{array}{l} R_2 \times \frac{1}{6} \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -2 & -2 \end{array}\right] R_3 + 2R_2 \rightarrow R_3 \left[\begin{array}{ccc|c} \boxed{1} & 0 & 3 & 5 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Pivots at "R_1, R_2"

$\Rightarrow$ So, x_3 is free.

$\Rightarrow$ $x_1 + 3x_3 = 5$

$x_1 = 5 - 3x_3$

$x_2 + x_3 = 1$

$x_2 = 1 - x_3$,

if $\begin{cases} x_3 = t \\ x_2 = 1 - t \\ x_1 = 5 - 3t \end{cases}$, $t \in \mathbb{R}$.

$\Rightarrow$ In conclusion, since we can solve for x_1, x_2, x_3, (although there are infinite many solutions), we can conclude that $\vec{b}$ is in span{columns of A}.

Remark: Not "unqiue" because $\vec{b}$ is in fact in span$\left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ 5 \\ 1 \end{pmatrix} \right\}$, and $\begin{pmatrix} 0 \\ 8 \\ -2 \end{pmatrix}$ is in fact in the same span as well (so it is extra!).

③ (3 pts) Let $\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$

Is span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \mathbb{R}^3$? why or why not?

YES Span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \mathbb{R}^3$ because

the matrix $\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 4 \\ 0 & -3 & -1 \\ -2 & 8 & -5 \end{bmatrix}$ has REF that

has a pivot in every row

(General Fact: For any $A : m \times n$

$A\vec{x} = \vec{b}$ is consistent for every $\vec{b} \in \mathbb{R}^n$

$\Leftrightarrow$ A has a pivot in every row

$\Leftrightarrow$ columns of A span $\mathbb{R}^m$).