

Practice Midterm 2

Problem 1: True/False (briefly explain your choice):

- a) The rank of a matrix A is equal to the number of rows with pivots in the row reduced echelon form of A .
- b) If $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ is a set of nonzero mutually orthogonal vectors, then they are linearly independent.
- c) The sum of two eigenvectors of a matrix A is again an eigenvector of A .
- d) If A is a 5×3 matrix and the columns of A are orthogonal, then the rows of A are also orthogonal.

Problem 2:

- a) Let $A = \begin{bmatrix} 3 & -2 & 8 \\ 0 & 5 & -8 \\ 0 & 0 & 3 \end{bmatrix}$. Determine if A is diagonalizable. If A is diagonalizable,

compute the matrices D and P and verify that $D = P^{-1}AP$

- b) Let $T : \mathbb{P}_3 \rightarrow \mathbb{P}_2$ be a map that takes $\mathbf{p}(t)$ to $\mathbf{p}'(t)$, i.e. T is the derivative. Show that T is linear and find the matrix of T relative to bases $\{1, t, t^2, t^3\}$ and $\{1, t, t^2\}$.

Problem 3:

- a) Let $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}$. Use the Gram-Schmidt process to produce an orthonormal

set of vectors out of the columns of A .

- b) Compute the QR-factorization of A .

Problem 4: Compute the equation of the line that best fits the data points $(2,3)$, $(3,2)$, $(5,1)$, $(6,0)$.