MIDTERM #2

① If $\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_n}\}$ is orthogonal then $\{\vec{v_1}, \dots, \vec{v_n}\}$ is linearly independent.

Ⓐ TRUE Ⓑ FALSE

② Let $\vec{u_1}, \vec{u_2}$ be eigenvectors of a matrix A and let λ_1, λ_2 be the corresponding eigenvalues. If $\vec{u_1}$ and $\vec{u_2}$ are linearly independent then $\lambda_1 \neq \lambda_2$.

Ⓐ TRUE Ⓑ FALSE

③ Same hypothesis as ②. If $\lambda_1 \neq \lambda_2$ then $\vec{u_1}$ and $\vec{u_2}$ are linearly independent.

Ⓐ TRUE Ⓑ FALSE

④ The matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ is diagonalizable.

Ⓐ TRUE Ⓑ FALSE.

⑤ Let A be a 5x5 matrix with a basis of eigenvectors $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}, \vec{v_5}\}$. Consider the matrix $Q = [\vec{v_1}\ \vec{v_2}\ \vec{v_3}\ \vec{v_4}\ \vec{v_5}]$. Which of the following matrices is <u>always</u> diagonal?

Ⓐ QAQ^{-1} Ⓑ Q^TAQ Ⓒ $Q^{-1}AQ$ Ⓓ None of these.

⑥ Is the set $\{1-t+t^2,\ 2-t^2,\ t+2t^2\}$ linearly independent?

Ⓐ YES Ⓑ NO

⑦ Let $\mathcal{E}$ denote the standard basis for $\mathbb{R}^2$ and $\mathcal{B} = \left\{ \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$.

The change of basis matrix $\underset{\mathcal{B} \leftarrow \mathcal{E}}{P}$ is

Ⓐ $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$ Ⓒ Neither A nor B.

⑧ Is the following set: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ orthonormal?

Ⓐ YES Ⓑ NO.

⑨ Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}$ be a basis for $\mathbb{R}^3$ and $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

The coordinate vector $[\vec{x}]_{\mathcal{B}}$ is

Ⓐ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$ Ⓒ $\begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$ Ⓓ None of These.

⑩ Let $\vec{x_1}, \vec{x_2}, \vec{x_3}$ be 3 linearly independent vectors in $\mathbb{R}^4$ and let $\vec{v_1}, \vec{v_2}, \vec{v_3}$ be orthogonal vectors coming from the Gram-Schmidt process.

Then: $\vec{v_1} = \vec{x_1}$ $\quad \vec{v_2} = \vec{x_2} - \dfrac{\vec{x_1}\cdot\vec{v_1}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1}$ $\quad \vec{v_3} = \vec{x_3} - \dfrac{\vec{x_2}\cdot\vec{x_1}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1} - \dfrac{\vec{x_2}\cdot\vec{v_2}}{\vec{v_2}\cdot\vec{v_2}}\vec{v_2}$

Ⓐ TRUE Ⓑ FALSE

(11) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 1 & 5 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$.

(11.1) One least squares solution $\hat{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ to the linear equation $A\vec{x} = \vec{b}$ is obtained by solving $A\vec{x} = \vec{c}$ where $\vec{c}$ is

(A) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ (C) $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

(11.2) The normal equation to $A\vec{x} = \vec{b}$ is:

$$\begin{aligned} 2x_1 + 12x_2 &= 4 \\ 12x_1 + 27x_2 &= 11 \end{aligned}$$

(A) TRUE (B) FALSE.

(12) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$: a basis for $\mathbb{R}^3$.

(12.1) The matrix $[A]_{\mathcal{B} \leftarrow \mathcal{B}}$ is:

(A) $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$.

(12.2) The characteristic polynomial for A is:

(A) $-\lambda(6-\lambda)^2$ (B) $\lambda^2(6-\lambda)$ (C) $(1-\lambda)(2-\lambda)(3-\lambda)$.

(13) Let A be an $m \times m$ matrix. Then $\text{Row}\,A = \text{Col}\,A$

(A) TRUE (B) FALSE.

(14) Let $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Is A diagonalizable? Why or why not?

(15) Let $T: \mathbb{P}_2 \to \mathbb{P}_2$ be $T(a_0 + a_1 t + a_2 t^2) = (a_0 + a_1) + (3a_2 - a_0)t + a_1 t^2$

and $\mathcal{B} = \{1, t, t^2\}$ be a basis for dom(T), $\mathcal{E} = \{1, 1+t, 1+t^2\}$ be a basis for Codomain (T). Compute $\underset{\mathcal{E} \leftarrow \mathcal{B}}{[T]}$ and $[T(1+t+t^2)]_{\mathcal{E}}$.

(16) Let $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \in \mathbb{R}^3$ and H be the plane $2x + y - 2z = 0$.

Compute $\text{proj}_H \vec{v}$ and $\text{dist}(\vec{v}, \text{proj}_H \vec{v})$.

(17) Let V be the vector space $\mathbb{R}^2$ along with the inner product defined as follows: Fix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, for any $\vec{x}, \vec{y} \in \mathbb{R}^2$

$$\langle \vec{x}, \vec{y} \rangle = A\vec{x} \cdot A\vec{y}.$$

In $(V, \langle , \rangle)$, use Gram-Schmidt on $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ to find an orthogonal basis.

(18) Solve the following initial-value problem:

$y''(x) + 2y'(x) + y(x) = 0 \qquad y(0) = 1, \quad y'(0) = 2.$