MATH 253: PROBLEM SET 2, DUE FRIDAY FEB 11

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(1). Let $\mathscr{A}$ be an abelian category and let $K(\mathscr{A})$ be as in class. Verify the octahedral axiom for $K(\mathscr{A})$ and complete the proof that $K(\mathscr{A})$ is a triangulated category.

(2). Give an example of an abelian category $\mathscr{A}$ for which $K(\mathscr{A})$ is not abelian.

(3). Let $\mathscr{A}$ be an abelian category, and let

$$0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

be a short exact sequence in $C(\mathscr{A})$. Recall from class that then we obtain a commutative diagram of complexes

$$\begin{array}{ccccccc} A & \xrightarrow{f} & B & \xrightarrow{\epsilon} & \mathrm{Cone}(f) & \xrightarrow{t} & A[1] \\ \downarrow{\scriptstyle \mathrm{id}_A} & & \downarrow{\scriptstyle \mathrm{id}_B} & & \downarrow{\scriptstyle h} & & \\ A & \xrightarrow{f} & B & \xrightarrow{g} & C & & \end{array}$$

where h induces an isomorphism

$$H^i(h) : H^i(\mathrm{Cone}(f)) \to H^i(C)$$

for every i. Show that the diagram

$$\begin{array}{ccc} H^i(\mathrm{Cone}(f)) & \xrightarrow{H^i(t)} & H^{i+1}(A) \\ \downarrow{\scriptstyle H^i(h)} & \nearrow{\scriptstyle \partial} & \\ H^i(C) & & \end{array}$$

commutes, where ∂ is the boundary map constructed in class using the snake lemma.