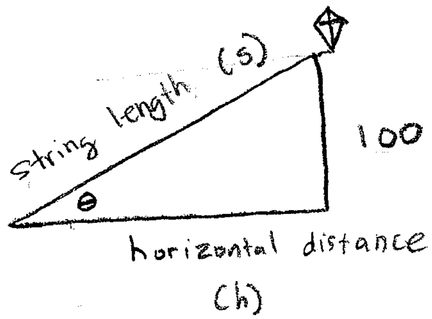
(2)



From the problem statement, we also know $\frac{dh}{dt} = 8$ ft/sec.

We want to relate the rate of change of angle, $\frac{d\theta}{dt}$, to what we know, $\frac{dh}{dt}$.

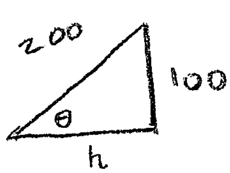
Let's choose an equation relating θ and h.

Notice that $\tan\theta = \frac{100}{h}$.

Then, $\frac{d}{dt}\tan\theta = \frac{d}{dt}\frac{100}{h}$

$$\sec^2\theta \frac{d\theta}{dt} = -\frac{100}{h^2}\frac{dh}{dt}$$

Since we are being asked for $\frac{d\theta}{dt}$ when $s = 200$ ft, we need to know what θ and h are at that time.



We have $\sin\theta = \frac{100}{200} = \frac{1}{2}$, so $\theta = \frac{\pi}{6}$ (remember your unit circle.)

Also, $h^2 + 100^2 = 200^2$, So $h = \sqrt{200^2 - 100^2}$
$= 100\sqrt{3}$

$\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

Thus $\sec^2\left(\frac{\pi}{6}\right)\frac{d\theta}{dt} = -\frac{100}{100\sqrt{3}}(8)$

$$\frac{d\theta}{dt} = -\frac{8}{\sqrt{3}} \cdot \frac{3}{4} = -2\sqrt{3}$$

Since the problem asked "how fast is θ decreasing" we may want to answer $2\sqrt{3}$ $\frac{\text{radians}}{\text{sec}}$.

6 (a) $\sqrt{99.8}$ is close to $\sqrt{100} = 10$.

To estimate $\sqrt{99.8}$, we find the equation of the tangent line to $f(x)=\sqrt{x}$ @ $x=100$ and calculate the line's value at $x = 99.8$.

Slope m of the tangent line is $f'(100)$.

$f'(x) = \frac{1}{2\sqrt{x}}$, $f'(100) = \frac{1}{20}$.

So $y - y_0 = m(x - x_0)$,

$$y - 10 = m(99.8 - 100)$$

$$y = 10 + \frac{1}{20}(-.2)$$

$$= 9.99.$$

So $\sqrt{99.8} \approx 9.99$

(A calculator shows 9.989995)

So, the "shortcut" formula is:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

where x_0 is the reference point (like $x_0 = 100$)