NS 22

1 a. $\int_0^2 2xe^{x^2}dx =$

Let $u = x^2$ $\quad u(0) = 0$

$du = 2x\,dx$ $\quad u(2) = 4$

So $\int_0^4 e^u du = e^4 - 1.$

b. $\int_1^4 4x\sqrt{3x^2+1}\,dx = I$

$u = 3x^2 + 1$ $\quad u(1) = 4$

$du = 6x\,dx$ $\quad u(4) = 49$

$I = \int_1^4 \frac{2}{3} \cdot 6x\sqrt{3x^2+1}\,dx = \frac{2}{3}\int_1^4 \sqrt{3x^2+1}\,6x\,dx$

$= \frac{2}{3}\int_4^{49} \sqrt{u}\,du = \frac{2}{3}\left[\frac{2}{3}u^{3/2}\right]_4^{49}$

$= \frac{4}{9}(7^3 - 2^3)$

c.

c. $$\lim_{R\to\infty} \int_{-R}^{R} \frac{e^{-x}}{1+e^{-x}}\,dx$$

Let's focus on the integral.

$$u = 1+e^{-x} \qquad du = -e^{-x}\,dx$$

$$u(-R) = 1+e^{+R} \qquad u(R) = 1+e^{-R}$$

$$\rightarrow \int_{-R}^{R} -\frac{(-e^{-x})}{1+e^{-x}}\,dx = -\int_{-R}^{R} \frac{-e^{-x}\,dx}{1+e^{-x}}$$

$$= -\int_{1+e^{R}}^{1+e^{-R}} \frac{du}{u} = -\Big[\ln|u|\Big]_{1+e^{R}}^{1+e^{-R}}$$

removed absolute value because everything on inside is positive.

$$= -\left[\ln(1+e^{-R}) - \ln(1+e^{R})\right]$$

$$\lim_{R\to\infty} \ln(1+e^{R}) - \ln(1+e^{-R}) = \lim_{R\to\infty} \ln\left(\frac{1+e^{R}}{1+e^{-R}}\right)$$

$$= +\infty$$

But! Suppose it was $\lim_{R\to\infty} \int_{0}^{R} \frac{e^{-x}}{1+e^{-x}}\,dx$

(skipping steps)

$$= \lim_{R\to\infty} -\Big[\ln|u|\Big]_{2}^{1+e^{-R}} = \lim_{R\to\infty} \ln 2 - \ln(1+e^{-R})$$

$$= \lim_{R\to\infty} \ln\left(\frac{2}{1+e^{-R}}\right) = \ln 2.$$

(d) $\displaystyle\int_1^{17} \frac{\ln x}{x}\,dx$ $\qquad u = \ln x \qquad u(1) = 0$

$\qquad du = \frac{1}{x}\,dx \qquad u(17) = \ln 17$

$$\int_0^{\ln 17} u\,du = \frac{1}{2}(\ln 17)^2$$

(e) $\displaystyle\int_{-\pi/2}^{\pi/2} \sqrt{1-\sin^2 x}\,\cos x\,dx$

technically $|\cos x|\cdot\cos x$ but $\cos x > 0$ on $(-\pi/2, \pi/2)$

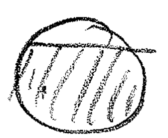
$$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos^2 x}\,\cos x\,dx = \int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{1+\cos 2x}{2}\,dx = \left[\frac{1}{2}x + \frac{\sin 2x}{4}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{\pi}{4} + 0 - -\frac{\pi}{4} + 0 = \frac{\pi}{2}$$

I bet you thought to do a u-substitution.

[Fun Fact: doing a "reverse u-substitution" method on $\int \sqrt{1-x^2}\,dx$ gives you the above integral, so you can calculate the area of stuff like $\int_{-1}^{a} \sqrt{1-x^2}\,dx$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