1 (a) $\lim_{n\to\infty} \underbrace{\frac{4}{n}}_{\Delta x} \sum_{i=1}^{n} \underbrace{\frac{4i}{n}}_{x_i} = \int_0^4 x\,dx$

(b) $\lim_{n\to\infty} \frac{\pi}{n} \sum_{i=1}^{n} \sin\left(\frac{\pi i}{n}\right) = \int_0^{\pi} \sin x\,dx$

(c) $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \frac{1}{\left[2+\left(\frac{2i}{n}\right)\right]^2} = \int_2^4 \frac{1}{x^2}\,dx$

or $= \int_0^2 \frac{1}{(2+x)^2}\,dx$

2. $f(x) = x^n \qquad [a,b] = [0,1]$

$\Delta x = \frac{b-a}{n} = \frac{1}{n} \qquad x_i = a + i\Delta x$

$= 0 + \frac{i}{n} = \frac{i}{n}$

$= \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} f\left(\frac{i}{k}\right) = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} \left(\frac{i}{k}\right)^n$

$= \lim_{k\to\infty} \frac{1}{k^{n+1}} \sum_{i=1}^{k} i^n$

If $n=1$:

$$\lim_{k\to\infty} \frac{1}{k^{1+1}} \sum_{i=1}^{k} i = \lim_{k\to\infty} \frac{\frac{1}{2}k(k+1)}{k^2} \overset{\lim_{k\to\infty}}{=} \frac{k^2+k}{2k^2}$$

$$= \lim_{k\to\infty} \frac{\frac{1}{k^2}\cdot\left[k^2+k\right]}{\frac{1}{k^2}\left[2k^2\right]} = \lim_{k\to\infty} \frac{1+\frac{1}{k}}{2} = \frac{1}{2}.$$

If $n=2$:

$$\lim_{k\to\infty} \frac{1}{k^3} \sum_{i=1}^{k} i^2 = \lim_{k\to\infty} \frac{k(k+1)(2k+1)}{6k^3}$$

$$\ldots = \frac{1}{3}$$

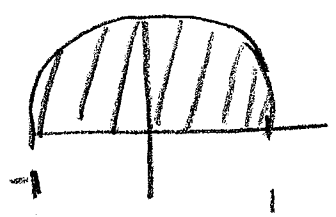
$n=3$:

$$\lim_{k\to\infty} \frac{1}{k^4} \sum_{i=1}^{k} i^3 = \lim_{k\to\infty} \frac{1}{k^4}\left(\frac{k(k+1)}{2}\right)^2$$

$$= \frac{1}{4}.$$

3. $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \sqrt{1-\left(-1+\frac{2i}{n}\right)^2}$

$$= \int_{-1}^{1} \sqrt{1-x^2}\,dx \quad \text{or} \quad \int_0^2 \sqrt{1-(x-1)^2}\,dx$$



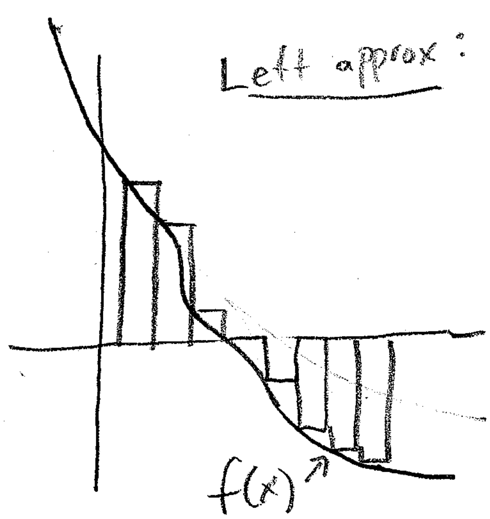
what is $y = \sqrt{1-x^2}$?

Well, $y^2 = 1-x^2 \Rightarrow x^2+y^2=1$.

semicircle

Area $= \pi(1)^2 \cdot \frac{1}{2} = \frac{\pi}{2}$.

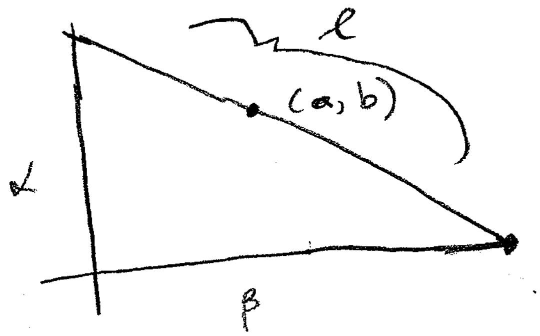
4. Intuitively : Warning: (This is not a proof.)



Always overestimates.
Don't be confused by the negative part. If we are estimating an integral, the rectangles under the x-axis have negative values.

(Now, if someone told you to estimate the area <u>between</u> the x axis & f(x), then we can't tell for sure.) if over or underest.

5.



Line through (a,b)

$$y = mx + c$$

$$b = ma + c$$

We use m, the slope, as the parameter we vary. We shall find the m-value so that the distance is minimized.

So $c = b - ma$.

Our line is $y = mx + b - ma$

β is the x coordinate when $y = 0$:

$$0 = m\beta + b - ma$$

$$\beta = \frac{ma - b}{m} \qquad \alpha = c = b - ma$$

$$\ell = \sqrt{\alpha^2 + \beta^2} \quad \text{So we minimize} \quad L = \ell^2 = (b - ma)^2 + \left(\frac{ma - b}{m}\right)^2$$

$$L' = 2\left(\frac{ma - b}{m}\right)\left(\frac{ma - (ma - b)}{m^2}\right) + 2(b - ma)(-a)$$

$$= \frac{(b - ma)(-2b - 2am^3)}{m^3}$$

$m = \frac{b}{a}$ Doesn't work since $m < 0$ to enclose a region,

$$-2b - 2am^3 = 0 \Rightarrow m^3 = -\frac{b}{a} \qquad m = \sqrt[3]{-\frac{b}{a}}$$

By the First Derivative Test it is a minimum.