

Worksheet 12
October 11th, 2007

1. If $Ca^x = 32$ at $x = 2$ and $Ca^x = \frac{1}{2}$ at $x = -1$. Find a and C .
2. How do the derivatives compare to each other? Do not use that $\frac{d}{dx} \ln(x) = \frac{1}{x}$.
 - (a) $\ln(x)$ and $\ln(100x)$
 - (b) $\ln(x^2)$ and $\ln(2x)$
 - (c) $\ln(ax^n)$ and $\ln(bx^m)$
3. Find the inverse of the following and state the domain and range
 - (a) $3x + 2$
 - (b) $x^3 + 1$
 - (c) $\frac{1}{x} + 2$
 - (d) e^{x^3}
 - (e) $\ln(2x^4)$
4. Derive the change of base formula $\log_b(x) = \frac{\ln(x)}{\ln(b)}$
5. Find the following derivative:
 - (a) $x^2 \ln(x)$
 - (b) $\ln |\sin(x)|$
 - (c) $e^{4 \ln(x)}$
 - (d) $e^{\sin^2(x)}$
6. Differentiate.
 - (a) $y = (x + 3)^5(x + 3)^{50}(x^2 + x + 3)^{20}(9x^3 + 2)^{11}$
 - (b) $y = (4x)^x$
7. Show that there is a point x such that $x^{x^2} = e^x$.

$$1 \quad f(x) = x^3 + 1$$

$$f^{-1}(x) = \sqrt[3]{x-1}$$

$$y = x^3 + 1$$

$$y - 1 = x^3$$

$$\sqrt[3]{y-1} = x$$

$$2 \quad \ln(1-x^2)$$

$$1-x^2 > 0$$

$$|x| < 1$$

if Range:

$$0 < 1-x^2 \leq 1$$

$$-\infty < \ln(1-x^2) \leq 0$$

$$(-\infty, 0]$$

$$3 \quad M(t) = C_0 \left(\frac{1}{2}\right)^{t/10}$$

$$10 = C_0 \left(\frac{1}{2}\right)^6$$

$$64 \cdot 10 = C_0$$

$$4 \quad f(x) = \ln[(\sin x)^{\sin x} \cdot (1 + \tan x)] = \sin x \ln(\sin x) + \ln(1 + \tan x)$$

$$f'(x) = \frac{\sin x \cos x}{\sin x} + \cos x \ln \sin x + \frac{\sec^2 x}{1 + \tan x}$$