Worksheet 16
April 4th, 2008

1. The following equation $Ax = b$ has no solution. Find the x which comes closest to b.

$$A = \begin{bmatrix} -2 & -7 \\ 1 & 5 \\ 1 & -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$$

2. For the following points (x_i, y_i, z_i), find a and b so that $\sum_{i=1}^{5}(z_i - ax_i - by_i)^2$ is minimized.

$$(2,3,1), (1,1,0), (-3,1,-4), (6,3,2), (1,0,0)$$

3. Using the inner product $\langle f(x), g(x)\rangle = \int_1^2 f(x)g(x)dx$, find an orthonormal basis of $\mathbb{P}^2$.

4. Call the basis you found in the previous exercise p_0, p_1, p_2. Suppose you had polynomials $f = a_0p_0 + a_1p_1 + a_2p_2$ and $g = b_0p_0 + b_1p_1 + b_2p_2$. Find a nice formula for $\int_1^2 f(x)g(x)dx$.

5. Use the orthogonal polynomials found in the previous exercise, find the polynomial of degree at most two that minimizes $\int_1^2 (log(x) - p(x))^2 dx$. If this gets too tedious, ask me for the value of a particular integral. If you ask one that I have computed I'll tell you the answer.

6. Show that the following inequality holds.

$$|\int_0^1 2\sin(x)e^{x^2}dx| \leq \sqrt{\int_0^1 4\sin^2(x)dx}\sqrt{\int_0^1 e^{2x^2}dx}$$

7. Show that the following is an inner product on $\mathbb{R}^n$, with A an invertible $n \times n$ matrix.

$$\langle x, y\rangle = A^TAx \cdot y$$

8. (Challenge Question) Let f be defined for polynomials of degree at most three as follows: $f(p(x)) = \int_1^2 \frac{p(x)}{x^2+1}dx$. Explain why there exists a special polynomial of degree at most three, call it $Q(x)$, such that for any p of degree at most 3,

$$\int_1^2 p(x)Q(x)dx = f(p(x))$$

Find $Q(x)$.