

Worksheet 15  
March 31, 2008

1. If  $Q$  is an orthonormal  $m \times n$  matrix, and  $b$  is a vector of length  $m$  in the column span of  $Q$ , how can you solve  $Qx = b$  quickly? (Also, how would you determine whether  $b$  is in  $\text{Col}(Q)$  or not?)
2. Suppose  $A$  is a  $m \times n$  matrix. Suppose  $x$  is in  $\text{Col}(A)^\perp$ . What is the value of  $A^T x$ ? Why?
3. Let  $W = \text{span}\{v_1, v_2, v_3\}$ . Find an orthogonal basis of  $W$ . Then convert it to an orthonormal basis.

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} \quad v_3 = \begin{bmatrix} 22 \\ -2 \\ 1 \end{bmatrix}$$

4. Find the QR factorization of the following matrices.

(a)

$$\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \end{bmatrix}$$

5. For an orthonormal matrix  $Q$ , we know that  $Q^T Q = I$ ; in other words,  $Q^T Qx = x$ . Interpret  $QQ^T$ .