Worksheet 13

π, 2008

1. Find the characteristic polynomial of this matrix.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

2. What is the characteristic polynomial of a rotation matrix?

3. Find the characteristic polynomial of the arbitrary 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

   Solve the equation equal to 0, and classify under what conditions does the matrix have 0, 1, or 2 eigenvalues.

4. Let

$$A = \begin{bmatrix} -4 & -7 \\ 2 & 5 \end{bmatrix}$$

   Find two linear independent eigenvectors, v_1 and v_2. Let $\mathcal{B}$ be the basis of $\mathbb{R}^2$ consisting of v_1, v_2. Find the change of basis $P_{\mathcal{B}\to\mathcal{I}}$, where we mean $\mathcal{I}$ to be the standard basis.

   Use this to diagonalize A.

5. In an ecosystem the population of owls and mice are completely intertwined. At the beginning of the nth month we find that the populations of mice $M_n = \frac{17}{10}M_{n-1} - \frac{3}{10}O_{n-1}$ when M_{n-1} and O_{n-1} are the mice and owl populations at the beginning of the previous month. Similarly we find that $O_n = \frac{4}{5}M_{n-1} + \frac{3}{10}O_{n-1}$.

   (a) If M_0 and O_0 are the initial populations, find an expression for M_n and O_n in terms of $M_0 = 7$ and $O_0 = 8$.

   (b) Find the long term ratio of Mice to Owls, if $M_0 = 7$ and $O_0 = 8$.

   (c) What would happen if $M_0 = 3$ and $O_0 = 12$?

6. Are the following pairs of matrices similar?

(a)

$$A = \begin{bmatrix} -4 & -7 \\ 2 & 5 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$$

(b)

$$C = \begin{bmatrix} -4 & -7 \\ 2 & 5 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

(c)

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$