

Practice for Midterm 2

April 13th, 2008

1. Orthogonally diagonalize the following matrix.

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

2. On \mathbb{P}_4 , define the following inner product. $\langle p, q \rangle = p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1) + p(2)q(2)$
- (a) Find an orthogonal basis of the subspace \mathbb{P}_2 .
- (b) Let $r(x) = x^4 - 2x^2 + x$. Find the polynomial $s(x) \in \mathbb{P}_2$ that minimizes $\langle s(x) - r(x), s(x) - r(x) \rangle$.
3. (a) Prove that if λ is an eigenvalue of A and A is invertible, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1} .
- (b) Prove that $\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$.
4. Find the x such that $\|b - Ax\|$ is as small as possible, where $A = QR$ (Q orthonormal),

$$Q = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} \quad R = \begin{bmatrix} 2 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$$