

# TRANSLATION OF *MICROLOCAL ANALYTIC SINGULARITIES*: CORRECTIONS

MICHAEL VANVALKENBURGH

I kept the same label numbers as the original (except for a few cases in Section 7—see below), so in what follows I mostly locate line numbers in terms of distance from a fixed label. Also, I mostly only write the corrected excerpt.

Throughout, I write  $h$  instead of  $\lambda^{-1}$

For the time being,  $\mathfrak{h}$  (and sometimes  $H$ ) is my substitute for Sjöstrand’s  $h$ . Ideally I would use a curly  $h$ , like  $\ell$ , but I don’t know how to make that symbol.

In the proof of Lemma 2.7, we have  $\frac{\partial^2}{\partial t^2}[\varphi(tz)]$  under the integral sign.

In the first sentence of Section 3: We are looking at real-*valued* quadratic forms, not to be confused with quadratic forms having real coefficients.

Two paragraphs before Prop. 3.3: “allows us to find real coordinates  $(t, s), t, s \in \mathbb{R}^k$ ”

In (3.1): “ $\mathcal{G}\mathcal{F}u(x; h)$ ”

Two equations after (4.1), Sjöstrand’s book is missing a  $\lambda$  (for us,  $h$ ) in the exponent.

Two paragraphs before (4.7): “if and only if  $F(x, y)$  has a saddle”

Proof of Theorem 4.2: “ $c(x, \theta; h) = e^{-\frac{ix\theta}{h}} B(\sigma_A(\cdot, \theta; h) e^{\frac{i(\cdot)\theta}{h}})$ ”

In (5.11): I wrote it as a double integral :)

In (5.12) and the following sentence: “ $\text{Im}(\varphi + \psi)$ ”

In (6.1), one might be tempted to rewrite “ $\varphi'_x = \alpha_\xi$ ” by changing the sign (see (6.6) and the proof of Prop. 7.2). However, it is correct as it stands, since he writes the complex conjugate of  $u$  in Def. 6.1. This is the same as the occasional confusion caused by the convention “ $z = x - i\xi$ ”.

In (6.7'), (7.1'), etc.: I write primes instead of large tildes because I think it looks better. Or maybe it was because I couldn’t get it to work with a tilde. I don’t remember.

Around (6.8): I made the references to (6.7) rather than to (6.7')—no big deal

In Section 7, his equation numbers differ from mine. This is because in the original the equation numbers skip from (7.5) straight to (7.8). Hence his final equation is (7.19) and mine is (7.17).

Proof of Lemma 8.4: “for  $0 \leq x < a$ ”

Proof of Theorem 8.3: in three places I changed  $\eta_0$  or  $\eta'_0$  to  $-\eta_0$  or  $-\eta'_0$ .

Proof of Theorem 8.3: I defined  $\tilde{\varphi}$  differently, and this caused a further change: the function  $x^n \mapsto \dots$  is different.

Proof of Theorem 8.7: I introduced the notation  $(x(t), \xi(t))$  for the Hamilton flow, to distinguish from the earlier use of the notation  $x_t$ .

In the bibliography, I now cite Hörmander’s ALPDO, Vol. I, rather than what used to be “a book in preparation by Hörmander”

In Section 11, I use the symbol “ $\cap$ ”, whereas in the original the notation is “ $\top$ ” superimposed over “ $\cap$ ”. I don’t know how to make that symbol.

Those are the only changes I can remember; there might have been a few more.

### **New for January 15, 2009:**

Section 6 is now titled “The Analytic Wavefront Set, Essentially Following Bros-Iagolnitzer”. I take “analytic wavefront set” to be the default terminology, with “analytic singular spectrum” as the secondary, whereas the original takes the opposite convention.

Proof of Prop. 6.2: It seems more accurate to say “By the results of Section 5”

Proof of Theorem 6.4: “=  $\int e^{\frac{i}{h}\varphi(x,\alpha)} b(x, \alpha; h) \overline{u(x)} dx$ ”

### **New for January 31, 2009:**

After (4.12), in the definition of  $\Gamma_t$ : It is ok as it stands in the original. At one point I used “ $|x - y| \leq \frac{r}{t^2}$ ”, etc., but I rechecked it, and it’s ok with “ $|x - y| \leq \frac{r}{t}$ ”. After all,  $t^n$  comes from  $dy \wedge d\theta \mapsto dy \wedge d\bar{y}$ , and  $t^{-2n}$  comes from the volume (for  $y \in \mathbb{C}^n!$ ). Of course the argument also works with  $r/t^2$ , or  $r/t^{103}$  for that matter.

**New for May 31, 2009:**

I improved my English in Section 1.

In the proof of Proposition 1.2, I am writing  $L(dz)$  instead of  $L(dx)$ .

I corrected my own typo after (1.11): “ $\Omega_t \subset \mathbb{C}^{2n}$ ”—it’s correct in Sjöstrand’s book.

**New for June 7, 2009:**

I improved my English in Section 2.

In (2.2), changed it to  $(N + 1)^{n/2}$  (correct in Sjöstrand’s book).

Before (2.11), I’m writing  $\Gamma = \mathbb{R}_{\tilde{x}}^n$ .

In (2.11), it’s  $I(h)$  (correct in Sjöstrand’s book).

After (2.14) but before “putting  $\tilde{h} = \frac{h}{2C_2}$ ”:  $\mathfrak{S}(h)$  has a  $(2\pi)^{-n}$  factor.

**New for June 10, 2009:**

In Section 3, when defining what it means to be a good contour,  $I_\Sigma(h)$  should only have  $u$  under the integral sign.

In the penultimate sentence of the first paragraph of the proof of Lemma 3.2, I’m emphasizing that everything is evaluated at 0 (or  $(0, 0)$ ).

**New for July 7, 2009:**

In the statement of Lemma 4.1: “If  $\sigma_A = 0$  in...”

In the proof of Lemma 4.1 I am writing “ $u(x, \cdot, \cdot; h)$ ”

In the proof of Lemma 4.1, in the expression for  $b$  I got “ $(-1)^j$ ”. At any rate, it is not important.

In the proof of Theorem 4.2: “ $\sigma_B(x, \theta; h)$ ”

In (4.13’) I am going back to Sjöstrand’s language, since the emphasis here, considering (4.13), is on the fact that  $\kappa$  is a diffeomorphism.

After (5.8) (end of the paragraph): “such that  $a d\alpha = \dots$ ”

**New for July 8, 2009:**

In the proof of Proposition 6.2, the expression for  $Au$  should have the term  $\overline{\varphi(\bar{y}, \bar{\alpha})}$  in the phase. This is also what appears in the expression for  $f$ , where Sjöstrand correctly writes it as  $\overline{\varphi(x, \beta)}$ ; I am just adding all the bars for emphasis.

In the proof of Proposition 6.2,  $Au$  should have  $h^{-3n/2}$  in front. This also appears in the expression for  $f$ .

**New for July 12, 2009:**

Conditions (7.1), (7.2), and (7.3) are inconsistent with (7.1'), (7.2'), and (7.3') as written in Sjöstrand's book. After all, we would have the following counterexample:

$$\varphi(x, y) = -x \cdot y + \frac{i}{2}(y - y_0)^2,$$

where  $x_0 = \eta_0 \in \mathbb{R}^n$  and  $y_0 \in \mathbb{R}^n$ . One option for a correction would be to replace (7.3) by

$$\det \operatorname{Im} \frac{\partial^2 \varphi}{\partial x \partial y}(x_0, y_0) \neq 0.$$

This still holds for the standard phase. On the other hand, in his paper "Function spaces associated to global I-Lagrangian manifolds," Sjöstrand keeps (7.3) (although only for quadratic forms), and I do see where he uses (7.3) but not (7.3'). Hence I have opted to simply omit the subscript "1" in (7.3') and write the new condition as:

$$\det \nabla_x \nabla_y \varphi(x_0, y_0) \neq 0.$$

I may change this in the future.

**New for July 13, 2009:**

In Sections 5 and 7, I now use the term "resolution of the identity." Earlier I occasionally used the term "partition of unity," but "ROI" is a direct translation and moreover is suggestive of functional analysis (re: POVMs), whereas "POU" is suggestive of cut-off functions.

In the statement of Lemma 7.5: I imagine he means "near  $(0, 0)$  in  $\mathbb{C}_z^n \times \mathbb{C}_w^k$ ."

In the statement of Lemma 7.5: "and that  $z = 0$  is a saddle for..."

In the proof of Lemma 7.5: " $|\operatorname{Im} z(w)|$ ."

In the proof of Lemma 7.5: I am using the letter " $H$ " to replace " $h$ ." I also did this in the proof of Theorem 2.8.

Sjöstrand's citation for Lemma 7.5 might be incorrect. In glancing through the "Melin-Sjöstrand, 1974" article, I didn't see anything resembling Lemma 7.5. Their other article is not available to me.

The penultimate sentence in the proof of Proposition 7.4: " $I^V u = \dots d\alpha$ ."

**New for July 17, 2009:**

Improved English in Section 9, for example a sentence in the proof of Theorem 9.1 is now translated as "We also denote a representative by  $a(t, x, \alpha; h)$ ." (Anyone have a better suggestion? He just means that we denote both the formal symbol and a representative by the same letter.)

Throughout I am now writing "formal classical" [analytic symbol] instead of the opposite ordering.

After Proposition 7.2: " $\frac{\partial \tilde{\varphi}}{\partial y}(\tilde{x}_0, y_0) = -\eta_0$ "

**New for July 21, 2009:**

Just some cosmetic changes on the first page.

**New for August 11, 2009:**

I remembered some changes I made over a year ago and added them to the undated list at the beginning of this note.

UCLA DEPARTMENT OF MATHEMATICS, LOS ANGELES, CA 90095-1555, USA  
*E-mail address:* `mvanvalk@ucla.edu`