Worksheet 9

Sections 306 and 310
MATH 54

September 20, 2018

Exercise 1. Let W be the union of the first and third quadrants of the plane. That is: let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$.

(a) If $\mathbf{u}$ is in W and c is any scalar, is $c\mathbf{u}$ in W? Why?

(b) Can you find specific vectors $\mathbf{u}$ and $\mathbf{v}$ in W such that their sum is not in W.

(c) Is W a vector space?

Exercise 2. For each of the following sets, either use an appropriate theorem to show that the given set is a vector space, or find an specific example to the contrary.

$$\left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 5r - 1 = s + 2t \right\}$$

$$\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{matrix} a + 3b = c \\ b + c + a = d \end{matrix} \right\}$$

c1

Exercise 3. Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Is $\mathbf{w}$ in Col A? Is it in Nul A?

Exercise 4. Determine which of the following sets are bases for $\mathbb{R}^3$. Justify your answers.

$$\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\begin{bmatrix}2\\-2\\1\end{bmatrix}, \begin{bmatrix}1\\-3\\2\end{bmatrix}, \begin{bmatrix}-7\\5\\4\end{bmatrix}$$

$$\begin{bmatrix}1\\2\\-3\end{bmatrix} \begin{bmatrix}-4\\-5\\6\end{bmatrix}$$

Discuss with your group: Do you think that a set of two vectors can form a basis for $\mathbb{R}^3$? Why or why not? (We will discuss the idea of dimension soon, get excited!!)

Exercise 5. Let W be a vector space. Use the axioms of a vector space to show that $0\mathbf{u} = \mathbf{0}$ for every vector u in W.