Worksheet 5

Sections 306 and 310
MATH 54

September 6, 2018

Exercise 1. Assume T is a linear transformation. Find the standard matrix of T.

- $T : \mathbb{R}^3 \to \mathbb{R}^2$, and $T(\mathbf{e_1}) = (1,3)$, $T(\mathbf{e_2}) = (4,-7)$, $T(\mathbf{e_3}) = (-4,5)$, where $\mathbf{e_1}$, $\mathbf{e_2}$, and $\mathbf{e_3}$ are the columns of the 3×3 identity matrix.

- $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the horizontal x_1- axis and then reflects points through the line $x_1 = x_2$.

- $T : \mathbb{R}^2 \to \mathbb{R}^3$ and $T(x_1, x_2) = (x_1 - x_2, -2x_1 + x_3, x_1)$.

As a group, choose one of these transformations and figure out if it is one-to-one and onto.

Exercise 2. If possible, compute each of $3C - E$, CB, EB. If any of these computations are impossible, briefly explain why.

$$B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Exercise 3. If a matrix B is 5×3 and the product AB is 2×3, what is the size of A?

Exercise 4. How many rows does B have is BC is a 3×4 matrix?

Exercise 5. Suppose the second column of B is all zeros. WHat can you say about the second column of AB?

Exercise 6. Find matrices A, B, C, such that $AB = AC$, yet $B \neq A$.