• We compute $T(\vec{e}_1)$ and $T(\vec{e}_2)$.

$T(\vec{e}_1) = T(1,0) = (1-0, -2+0, 1) = (1,-2,1)$.

$T(\vec{e}_2) = T(0,1) = (0-1, 0+1, 0) = (-1,1,0)$.

So the standard matrix is $A = [T(\vec{e}_1), T(\vec{e}_2)] = \begin{bmatrix} 1 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix}$

Worksheet 5

Sections 306 and 310
MATH 54

September 6, 2018

Exercise 1. Assume T is a linear transformation. Find the standard matrix of T.

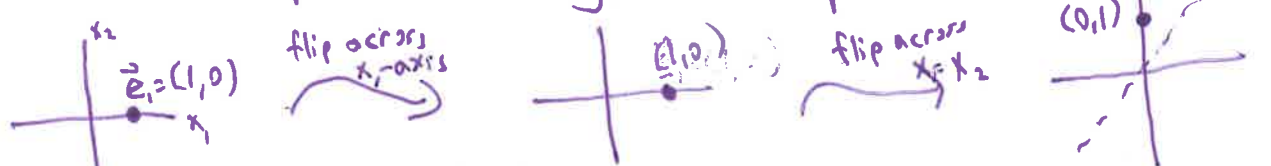
- $T : \mathbb{R}^3 \to \mathbb{R}^2$, and $T(\mathbf{e_1}) = (1,3)$, $T(\mathbf{e_2}) = (4,-7)$, $T(\mathbf{e_3}) = (-4,5)$, where $\mathbf{e_1}$, $\mathbf{e_2}$, and $\mathbf{e_3}$ are the columns of the 3×3 identity matrix.
- $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the horizontal x_1- axis and then reflects points through the line $x_1 = x_2$.
- $T : \mathbb{R}^2 \to \mathbb{R}^3$ and $T(x_1, x_2) = (x_1 - x_2, -2x_1 + x_2, x_1)$.

As a group, choose one of these transformations and figure out if it is one-to-one and onto.

• Since $T(\vec{e}_1) = (1,3)$, $T(\vec{e}_2) = (4,-7)$, $T(\vec{e}_3) = (-4,5)$, the standard matrix is:

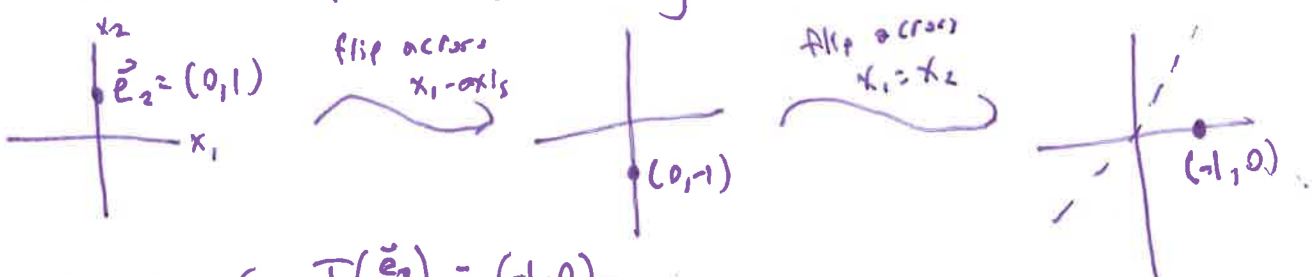
$$A = [T(\vec{e}_1)\ T(\vec{e}_2)\ T(\vec{e}_3)] = \begin{bmatrix} 1 & 4 & -4 \\ 3 & -7 & 5 \end{bmatrix}$$

• We first compute $T(\vec{e}_1)$ using the following picture:



So $T(\vec{e}_1) = (0,1)$.

We now compute $T(\vec{e}_2)$ using the following picture:



So $T(\vec{e}_2) = (-1,0)$.

So the standard matrix is $A = [T(\vec{e}_1), T(\vec{e}_2)] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Exercise 2. If possible, compute each of $3C - E$, CB, EB. If any of these computations are impossible, briefly explain why.

$$B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

- You can't subtract a 2×1 matrix from a 2×2 matrix. For addition/subtraction, dimensions must match.

- $CB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 7+2 & -5-8 & 1-6 \\ -14+1 & 10-4 & -2-3 \end{bmatrix} = \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$

- You can't multiply a 2×1 matrix with 2×3 matrix. since the circled numbers must match.

Exercise 3. If a matrix B is 5×3 and the product AB is 2×3, what is the size of A?

☆ An $a\times b$ times a $b\times c$ matrix is an $a\times c$ matrix.

So if B is 5×3 and AB is 2×3, A must be 2×5.

Exercise 4. How many rows does B have is BC is a 3×4 matrix?

From the fact above (marked with a ☆), The number of rows of BC is the same as the number of rows of B. so B must have 3 rows.

Exercise 5. Suppose the second column of B is all zeros. WHat can you say about the second column of AB?

The second column of AB must be all 0s. One way to see this is that the second column of AB can be written a a linear combination of the columns of A using the entries of the second column of B as the scalars. Since all these scalars are 0, the second column of AB must be the 0-vector.

Exercise 6. Find matrices A, B, C, such that $AB = AC$, yet $B \neq C$.

One example: Let A be the 0-matrix, and B, C be any non-equal matrices. Then

$$0B = 0C = 0, \quad \text{but } B \neq C.$$

Another example: Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

$$AB = AC = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \quad \text{but } B \neq C.$$