Worksheet 4

Sections 306 and 310
MATH 54

September 4, 2018

Exercise 1. Determine if each set of matrices is linearly independent.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 343 \\ 454 \\ 55 \\ -45 \\ 67 \end{bmatrix}$$

Exercise 2. For which values of h are the following vectors linearly dependent? Justify your answer!

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix} \quad \begin{bmatrix} 2 \\ -9 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

Exercise 3. Determine the possible row echelon forms of a 2×2 matrix with linearly dependent columns.

Exercise 4. For each pair $T, \mathbf{b}$, find a vector whose image under T is $\mathbf{b}$. Is this vector unique?

$$T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

Exercise 5. Describe geometrically what the following linear transformation T does. It may be helpful to plot a few points and their images!

$$T = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$$

Exercise 6. Let $\mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y_1} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$ and $\mathbf{y_2} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{e_1}$ to $\mathbf{y_1}$ and $\mathbf{e_2}$ to $\mathbf{y_2}$. What is the image of $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$?

Exercise 7. Show that $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$ is a linear transformation.